Raytracing (Part 2)

Slides by Sarah Jobalia and Kevin Li, with modifications by Kate Baker

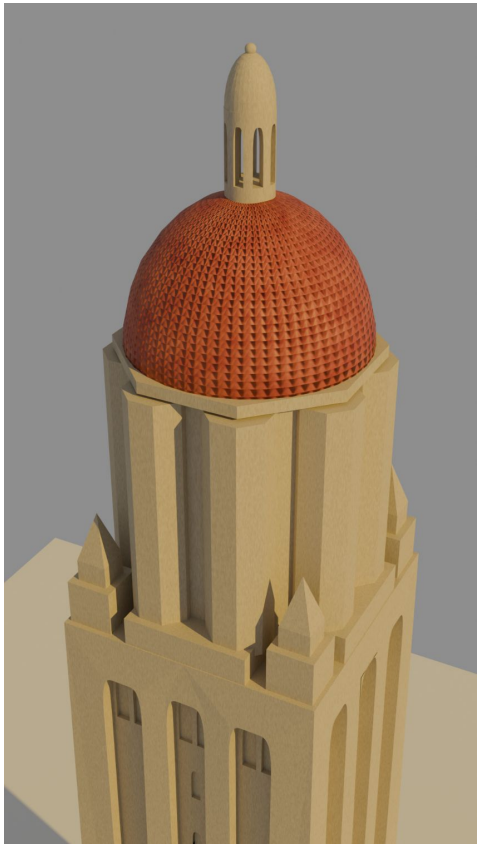
Housekeeping

- HW3 released, due Thursday
- Fill out the final presentation absence form!
- Thank you all for submitting HRCF feedback!
- It’s okay (and expected!) to be a bit confused about the derivations during lecture
- Guest speakers… stay tuned :)

Custom Texture Student Highlights!

Rio Suzuki

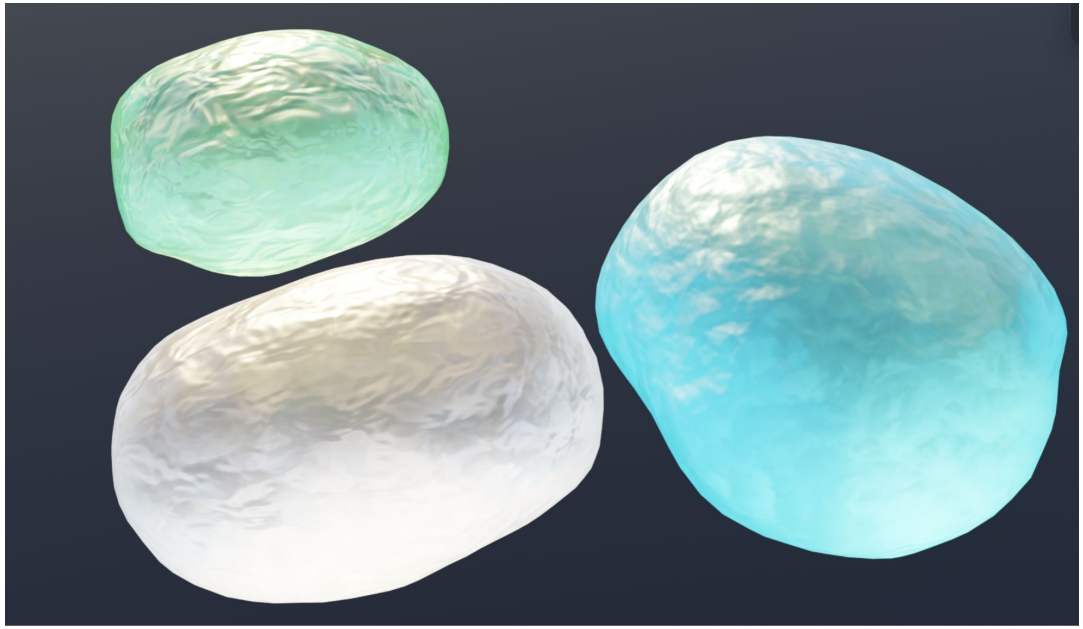
Ji Won Ryoo



Andrea Di Matteo

Custom Texture Student Highlights!

Silvia Festa



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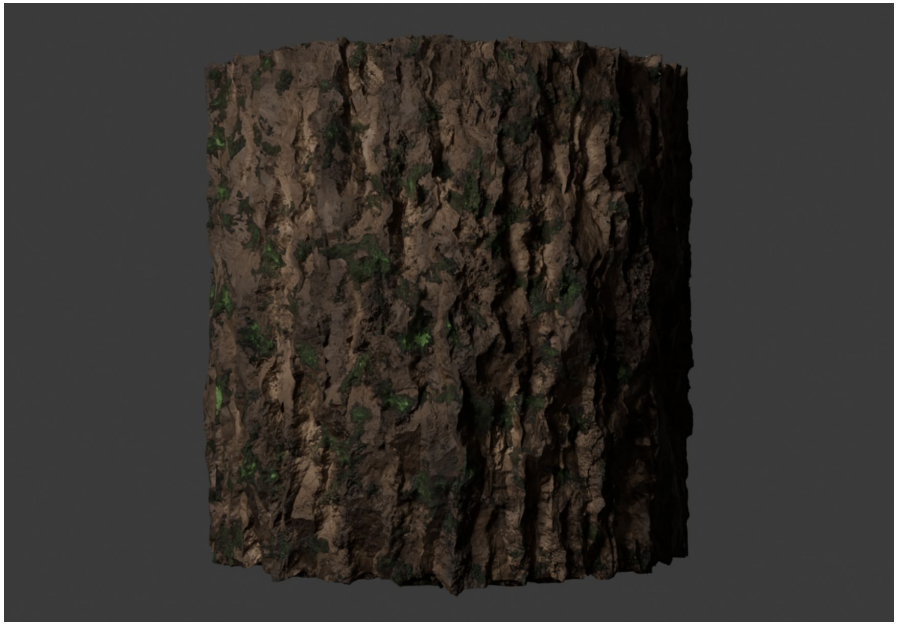
Siole Mayeski



Ya Wen

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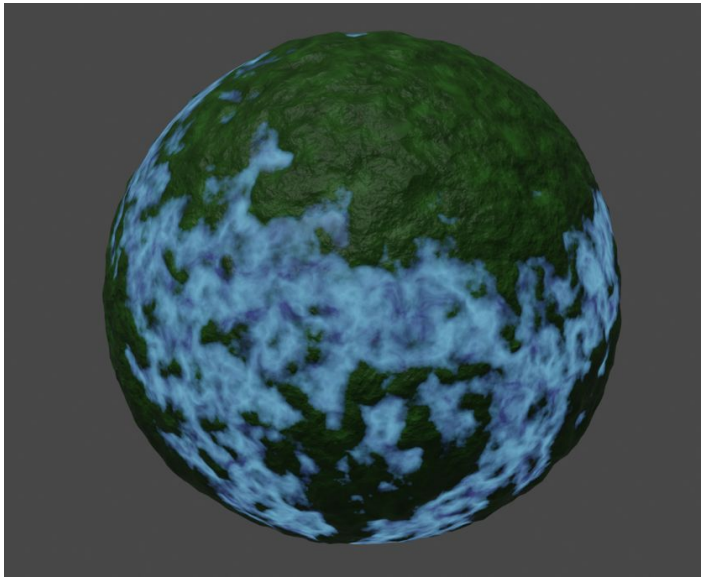
Jiuxing Chen



Daniel Li

Revisiting Questions

- In direct 3D ray-object intersection, how do we know if we've even hit the triangle that we care about?

- What if we can't solve this^ matrix?

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Recall: Ray-Triangle Intersection

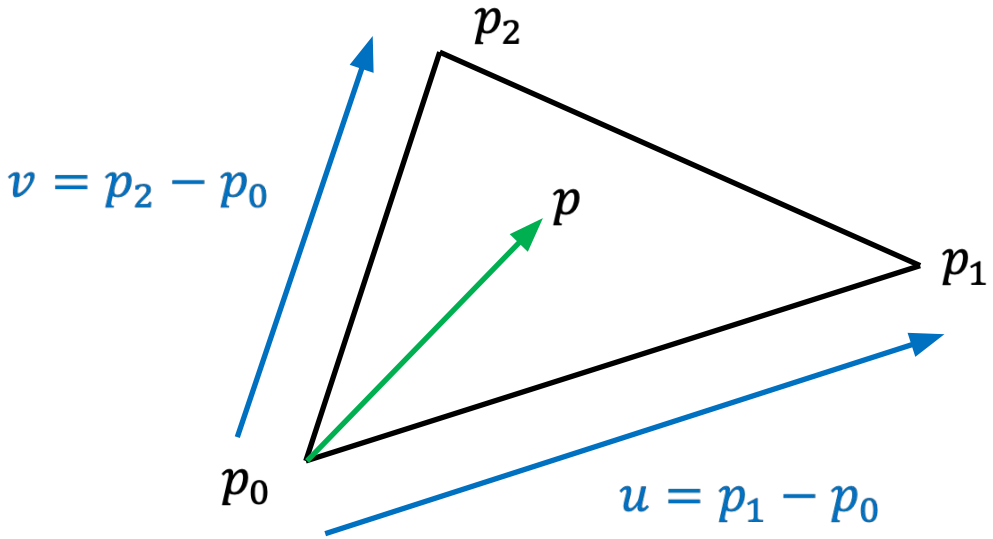
- Recall: most of our object will be **triangle meshes**
  - We need to intersect our ray with a triangle
  - Triangles are **planar** (3 points are guaranteed to make a plane)

- One technique:
  - ~~1) Determine how a ray intersects a **plane** for an intersection point~~ (Done!)
  - ~~2) Then, check if this intersection point is **inside the triangle**~~
- Another approach:
  - Consider 3D ray-object intersection directly

Recall: Triangle Basis Vectors

- Any point inside a triangle can be written as a sum of **one of the vertices + scalings of the edge vectors**

$$p = p_o + \beta_1 u + \beta_2 v \quad \text{with} \quad \beta_1, \beta_2 \in [0, 1], \beta_1 + \beta_2 \leq 1$$



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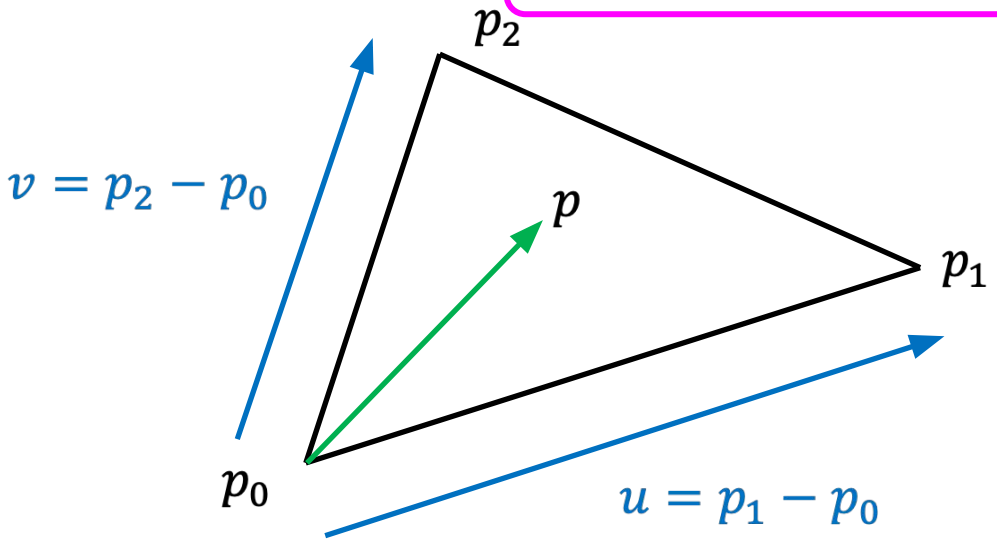

Revisiting Questions

- In direct 3D ray-object intersection, how do we know if we've even hit the triangle that we care about?

- What if we can't solve this^ matrix?

Recall: Direct Ray-Triangle Intersection

- A point inside a triangle is given by: $p = p_o + \beta_1 u + \beta_2 v$
- Substitute our ray: $R(t) = A + (P - A)t$

$$A + (P - A)t = p_o + \beta_1 u + \beta_2 v$$

$$(u, v, A - P) \begin{pmatrix} \beta_1 \\ \beta_2 \\ t \end{pmatrix} = A - p_o$$

- Solve matrix equation for: $\beta_1, \beta_2 \in [0, 1], \beta_1 + \beta_2 \leq 1$
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If determinant of matrix is nonzero: a solution!

Lecture Outline

- Ray tracing review
- Raycasting & recursive raytracing
- Reflected & transmitted rays
  - Refraction
  - Total internal reflection
- Attenuation & caustics

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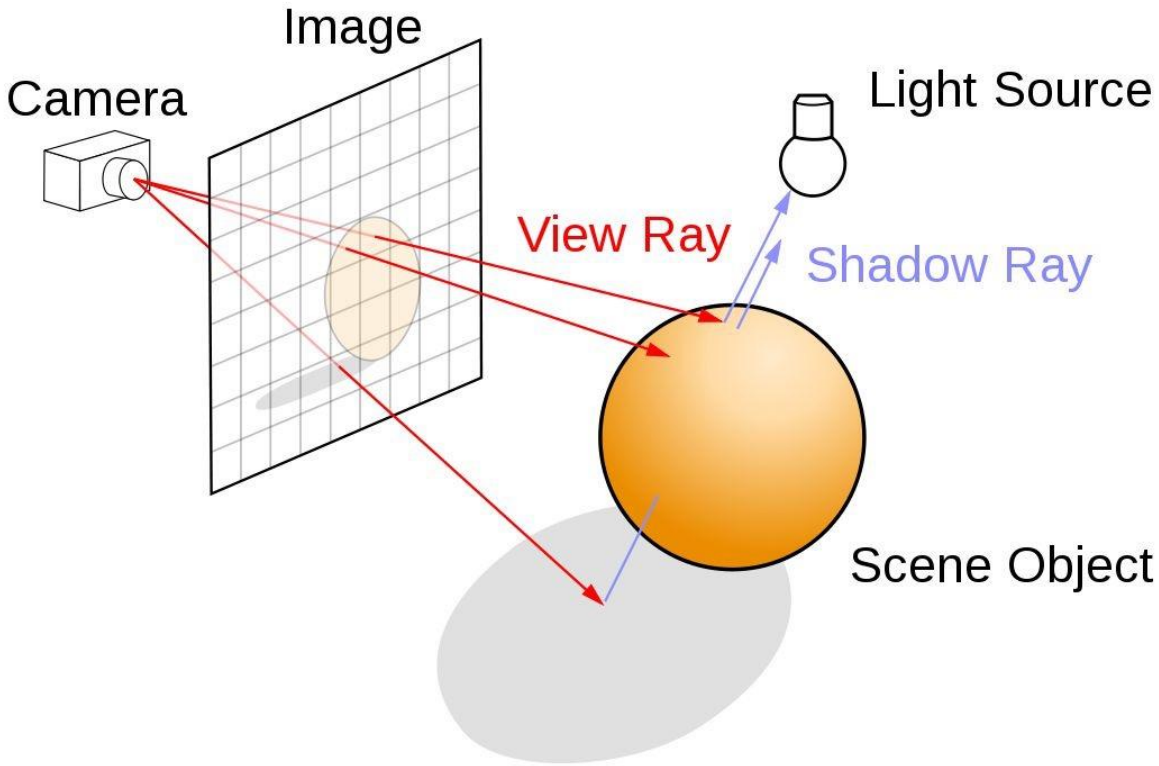
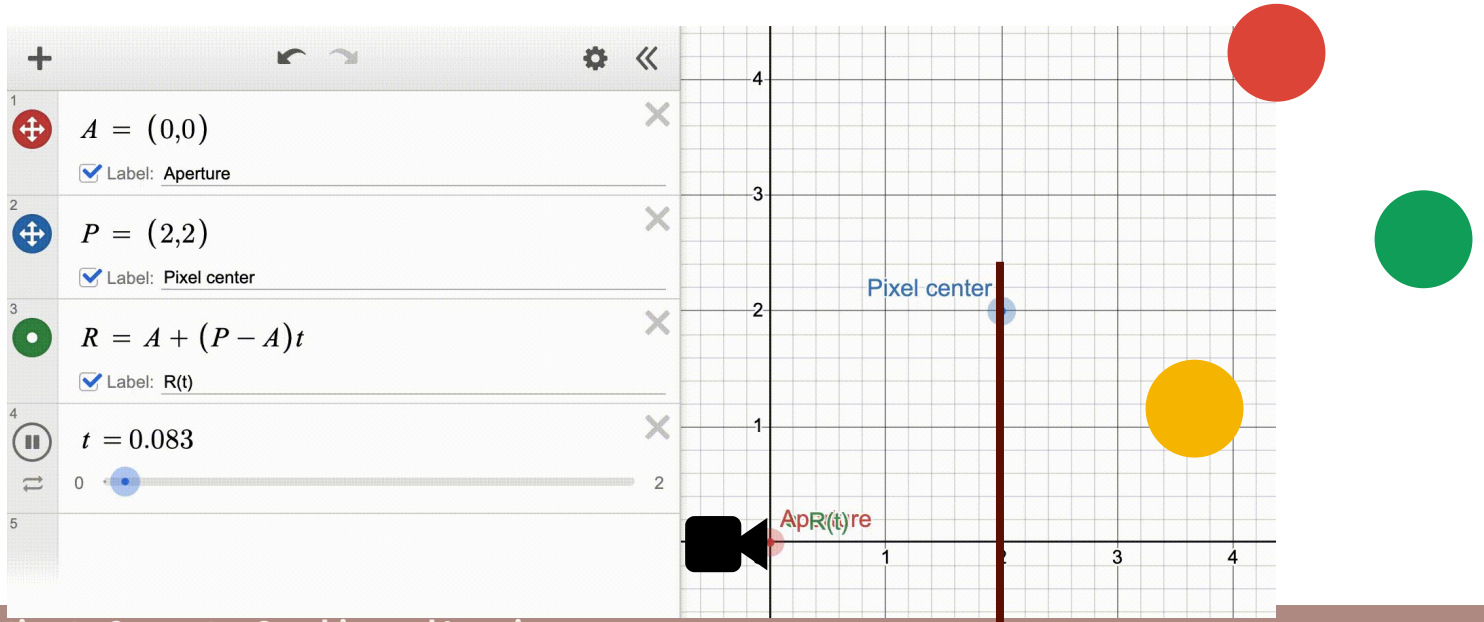


Raytracing Review

- For each pixel, shoot a camera ray
- Find the closest intersection of the ray with scene geometry
- Once we've found the intersection point, shoot shadow rays to all the light sources
- Compute pixel color by evaluating the BRDF (the lighting equation)



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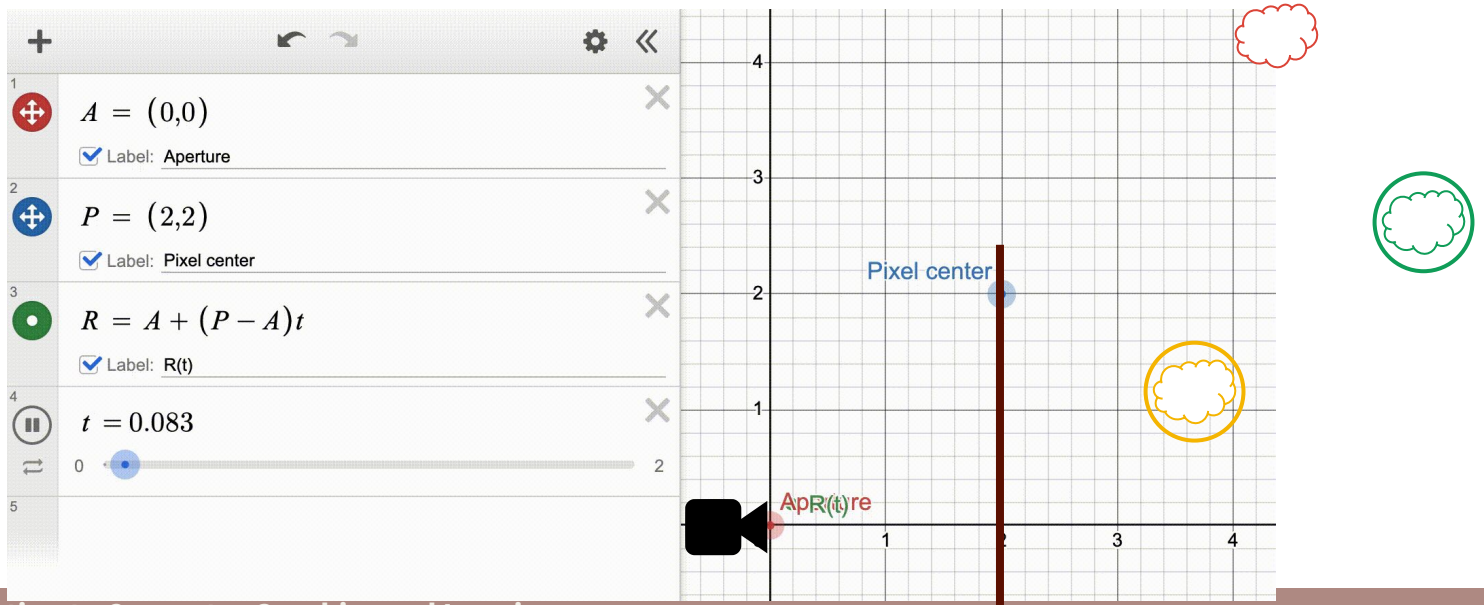

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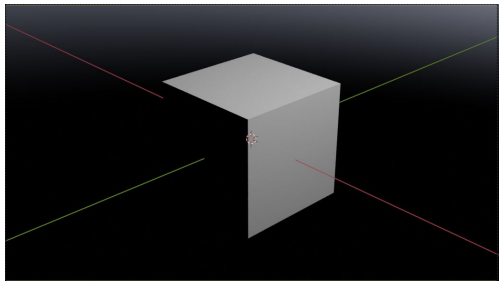

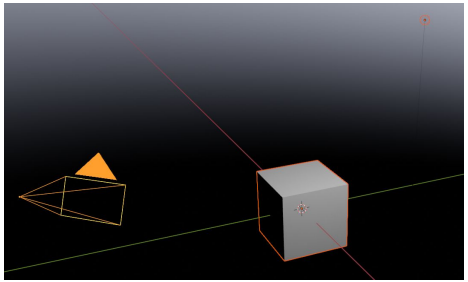

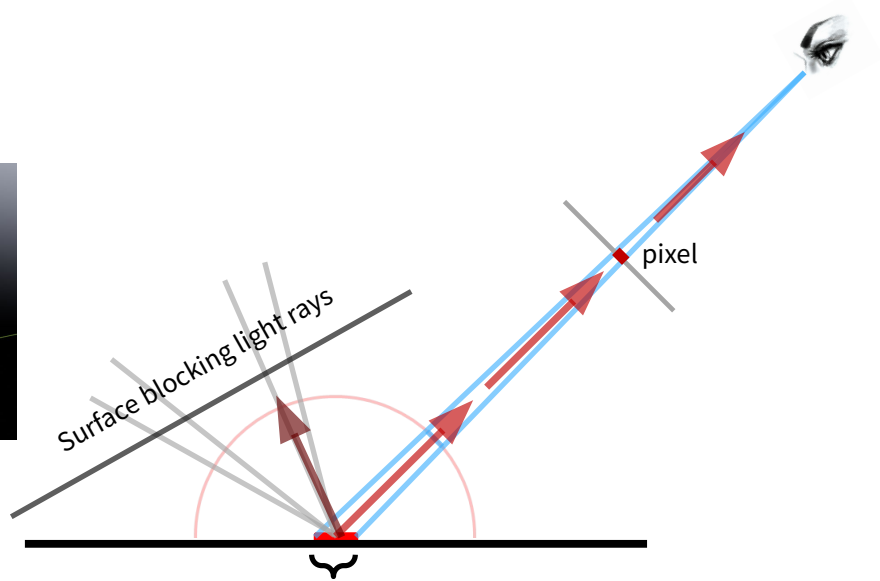

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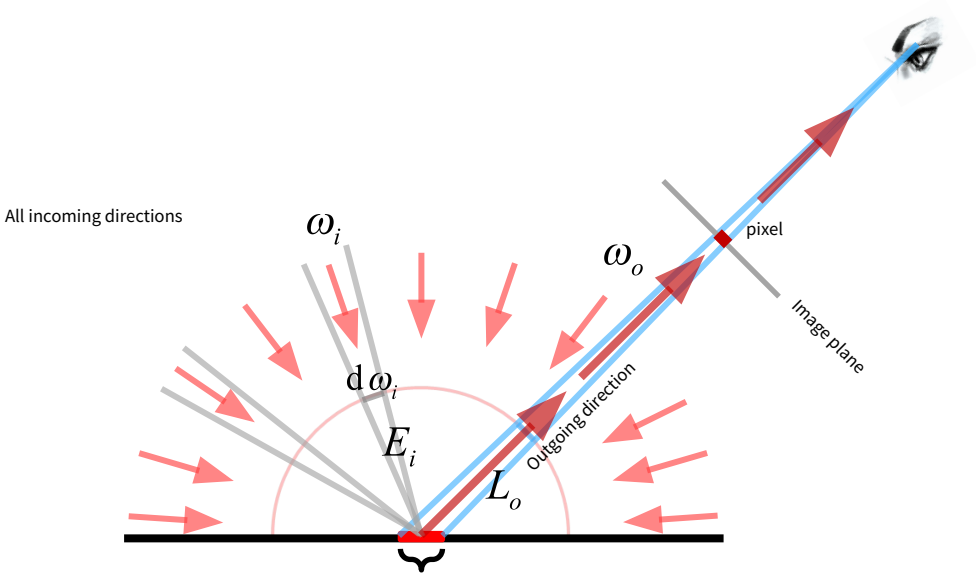

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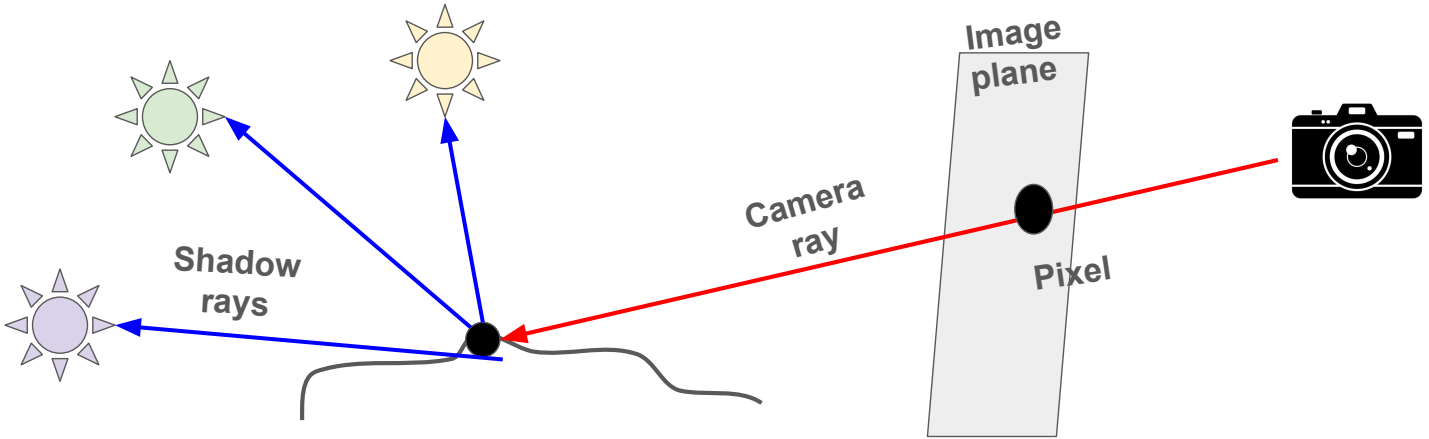
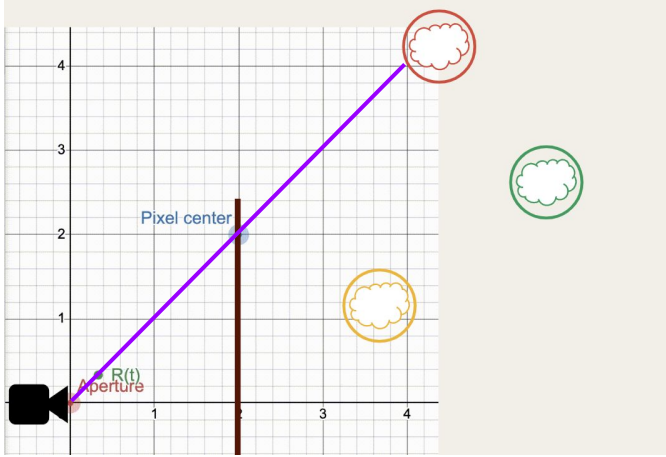


Raycasting

```
function generate_image():
    for pixel_position on film_plane:
        camera_ray=generate_ray_through_pixel(pixel_position)
        color=trace(camera_ray, objects, lights)
        film[pixel_position]=color
```

```
function trace(ray, objects, lights):
    intersection=ray_geometry_intersection(ray, objects)
    color=evaluate_rendering_equation(intersection, objects, lights)
    return color
```




```
function evaluate_rendering_equation(intersection, objects, lights):
    color=intersection.ambient
    for light in lights:
        shadow_ray=generate_ray_to_light(intersection, light)
        if does_not_hit_geometry(shadow_ray, objects):
            color+=BRDF(intersection.camera_ray,shadow_ray)*light.color*cos(theta)
    return color
```

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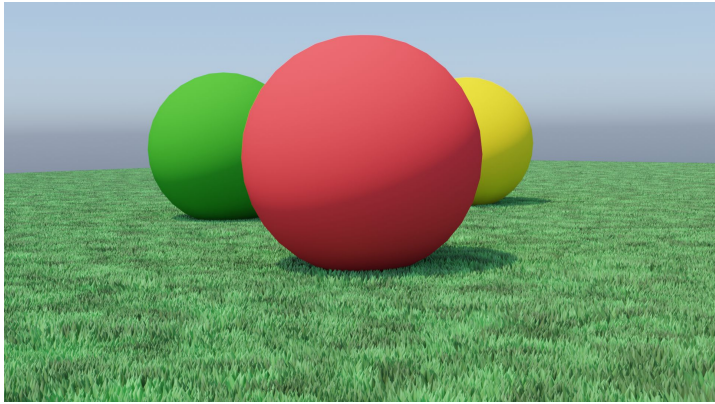
  - Total internal reflection
- Attenuation & caustics

Raycasting

- For each pixel, shoot a camera ray
- Find the closest intersection of the camera ray with scene geometry
- Shoot shadow rays to all the light sources
- Compute pixel color by evaluating the BRDF



Raycasting

- For each pixel, shoot a camera ray
- Find the closest intersection of the camera ray with scene geometry
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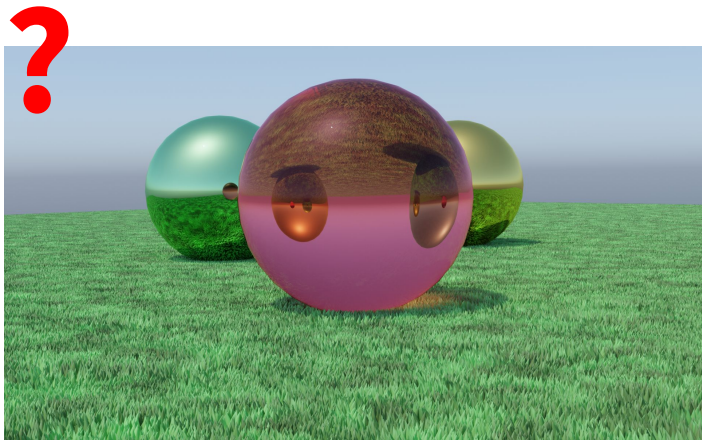
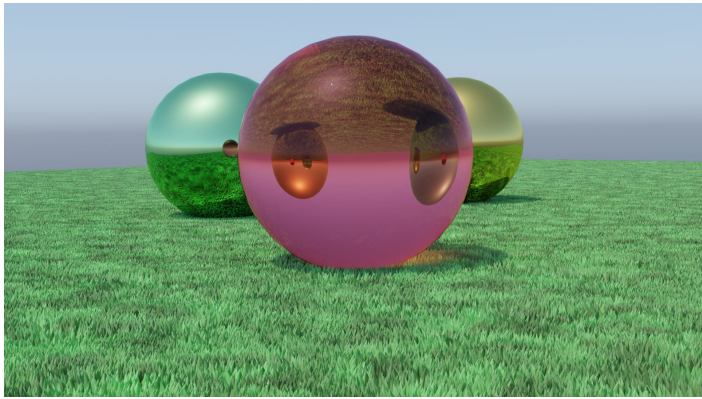
- Compute pixel color by evaluating the BRDF



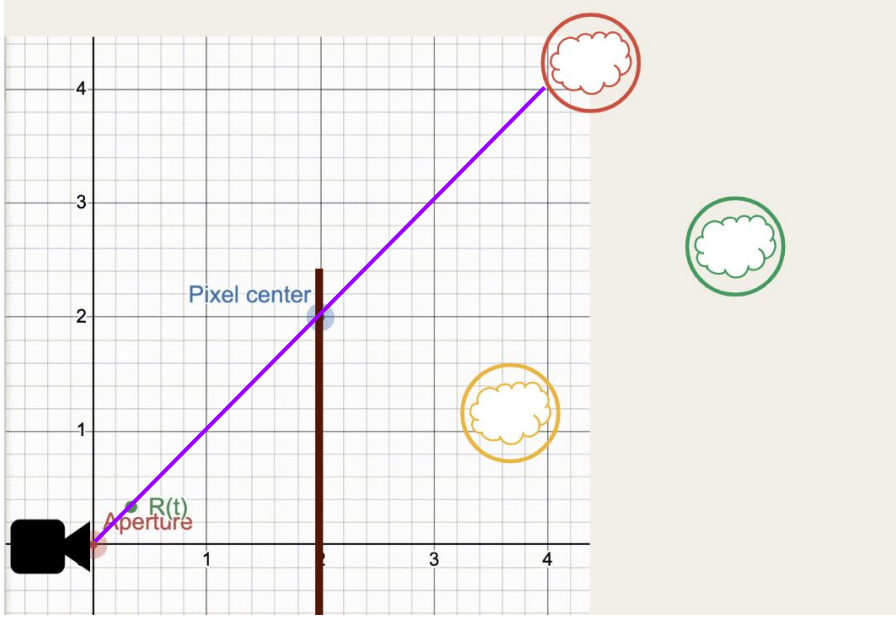
Raycasting

- For each pixel, shoot a camera ray
- Find the closest intersection of the camera ray with scene geometry
- Shoot shadow rays to all the light sources
- Shoot out new **transmission rays**
- Shoot out new **reflection rays**
- Compute pixel color by evaluating the BRDF



Recursive Raytracing

function trace(ray, objects, lights):
    intersection=ray_geometry_intersection(ray, objects)
    color=**compute_BRDF_at_intersection**(intersection, objects, lights)
    return color



Recursive Raytracing

function trace(ray, objects, lights):

intersection=ray_geometry_intersection(ray, objects)

color=**compute_BRDF_at_intersection**(intersection, objects, lights)

if (material at intersection has reflectivity>0)

reflected_ray=create_reflected_ray(ray, intersection)

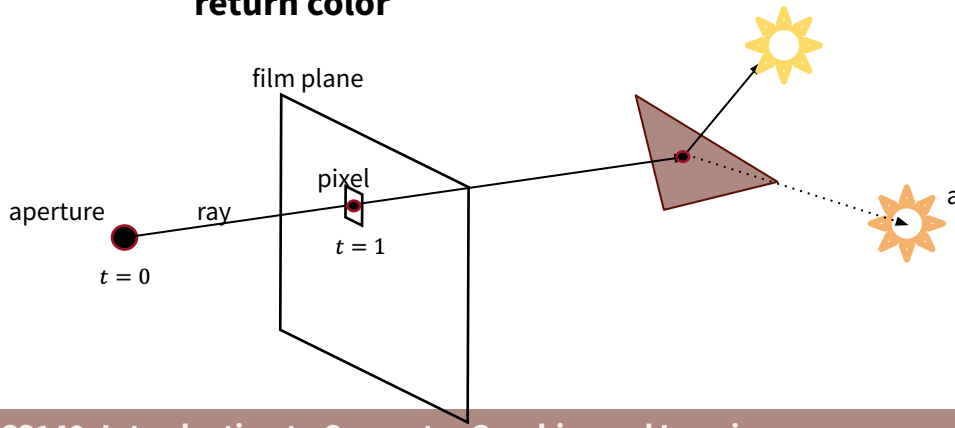
color=color+**trace(reflected_ray,objects,lights)*reflectivity**

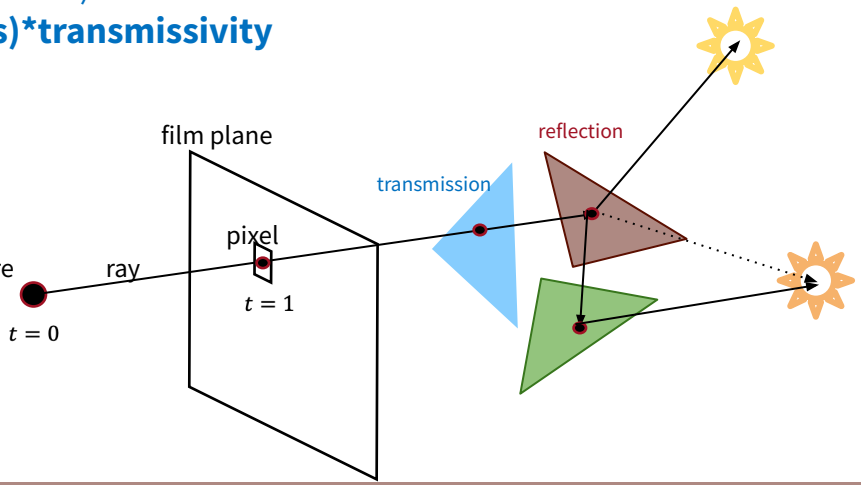
if (material at intersection has transmissivity>0)

refracted_ray=create _refracted_ray(ray, intersection)

color=color+**trace(refracted_ray,objects,lights)*transmissivity**

return color



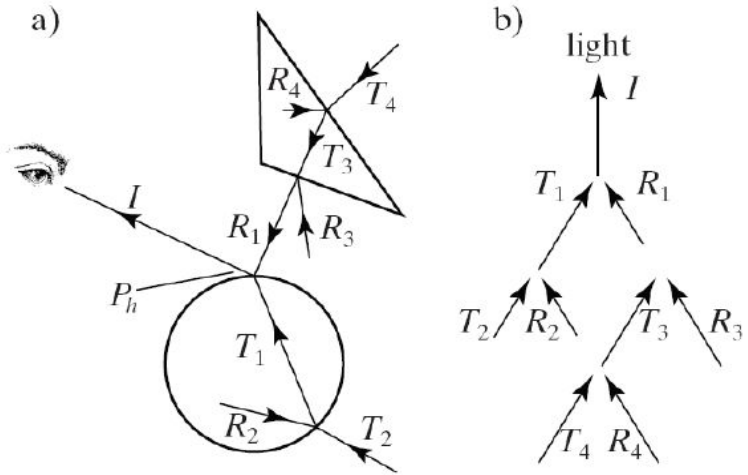


Recursive Raytracing

- How do we avoid infinite recursion (stack overflow)?
  - E.g. what if a transmission/reflection ray hit another transmissive/reflective object? And then another transmissive/reflective object? And then another…

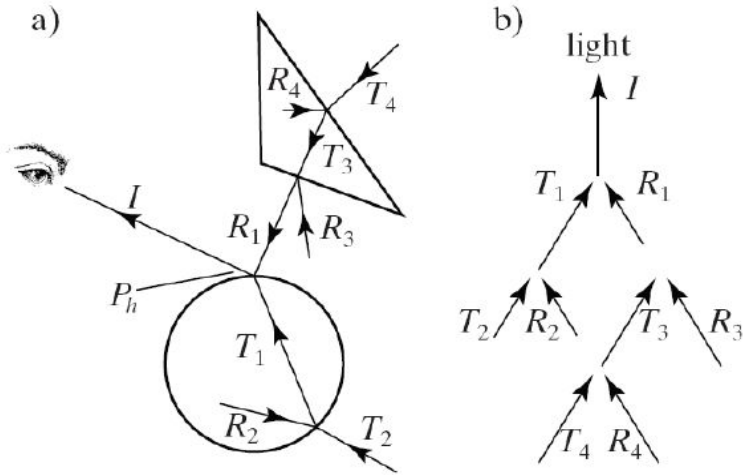
Recursive Raytracing: Termination

- To avoid infinite recursion (stack overflow), we need to keep track of how many bounces it's been
- Terminate raytracing after certain recursion depth



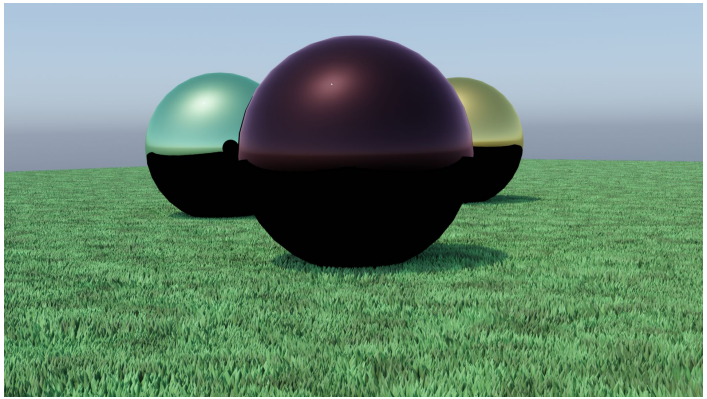
Recursive Raytracing: Termination

- To avoid infinite recursion (stack overflow), we need to keep track of how many bounces it's been
- Terminate raytracing after certain recursion depth
- This can happen with other materials besides mirrors
  - E.g. total internal reflection (later today)

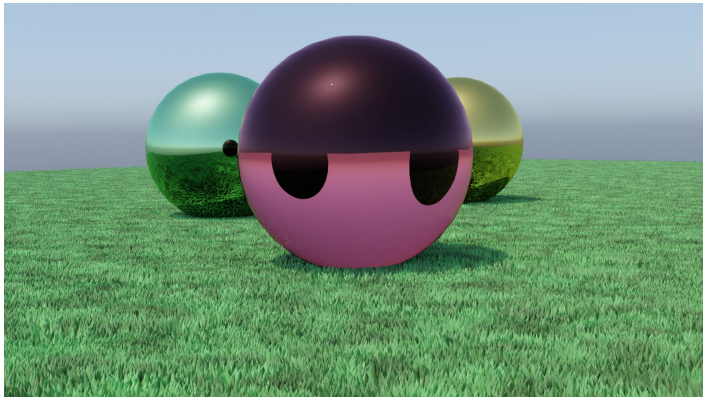


Recursive Raytracing

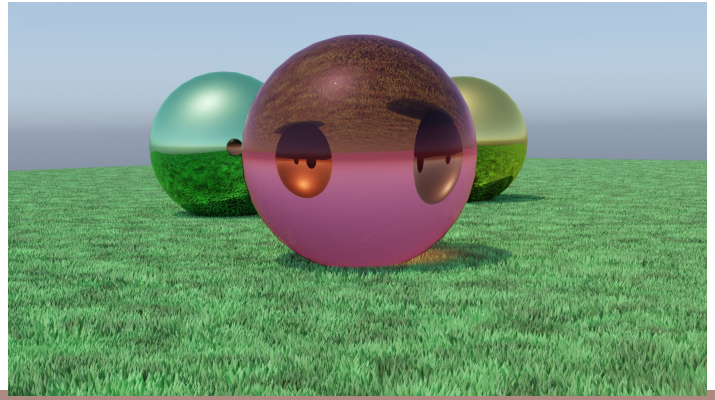
No recursions



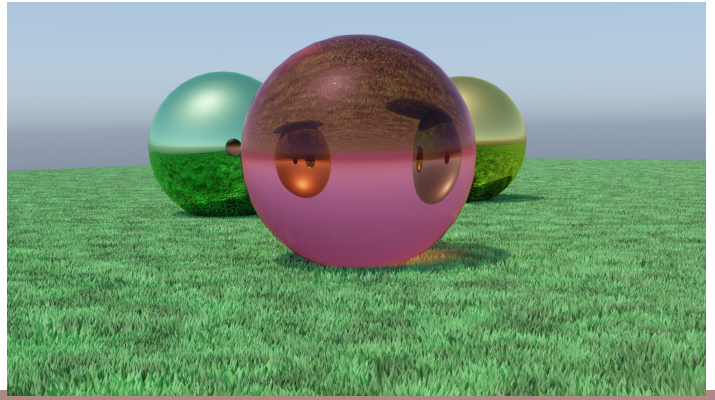
1 recursion bounce



2 recursion bounces



3 recursion bounces



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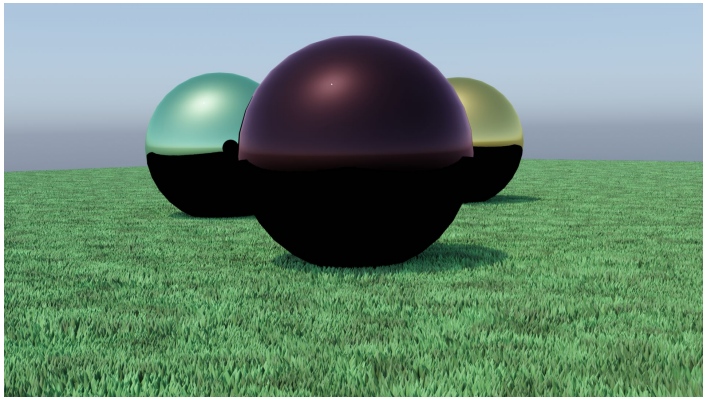

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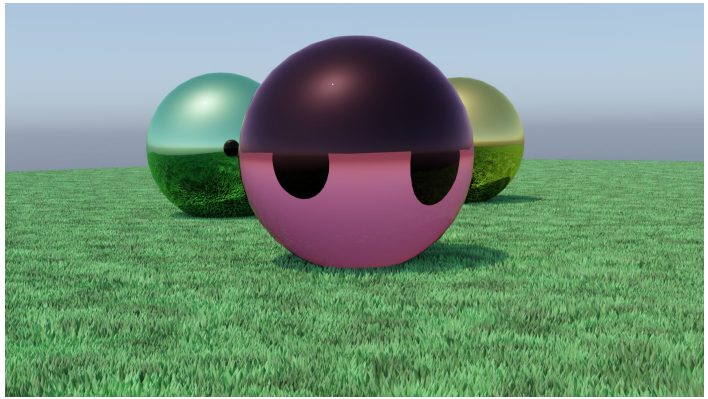

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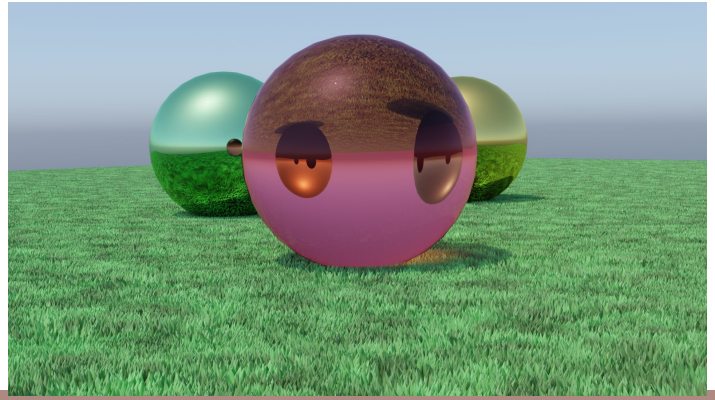

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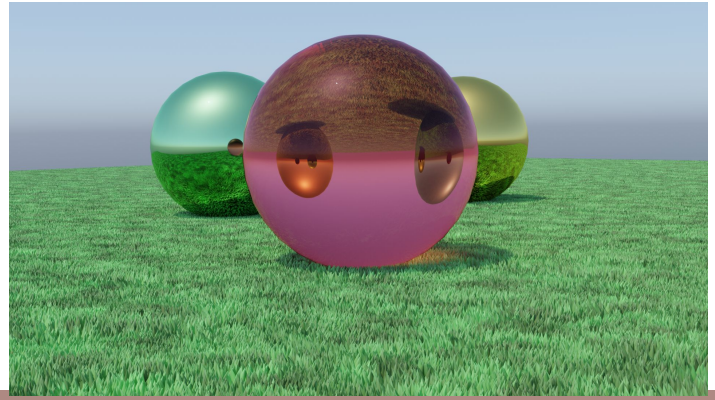

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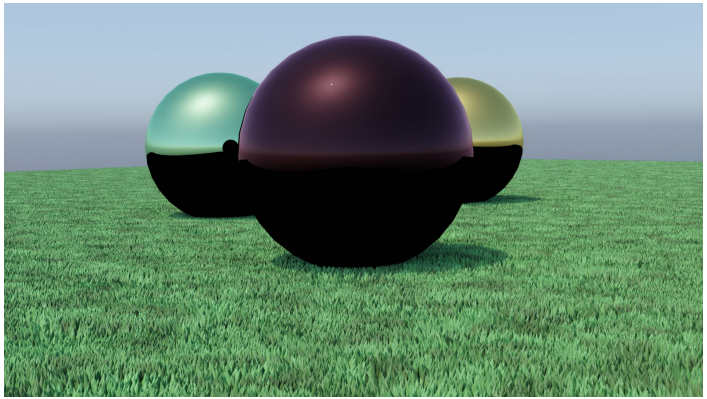

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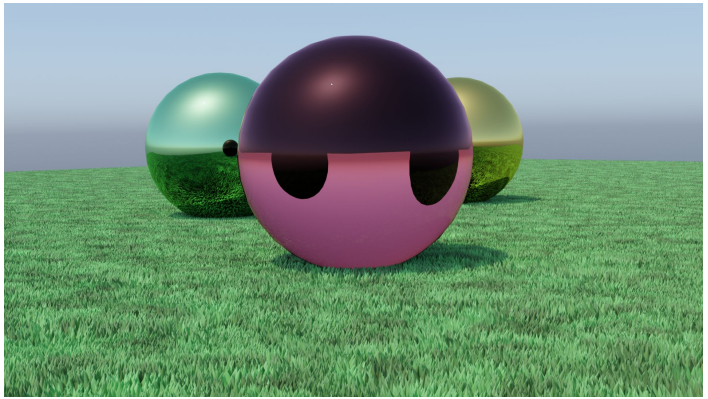

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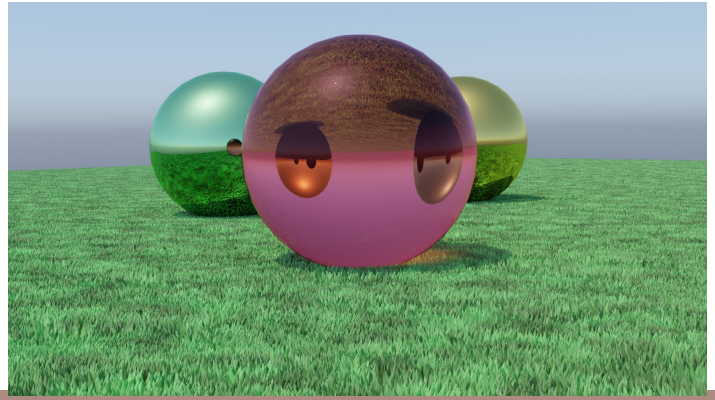

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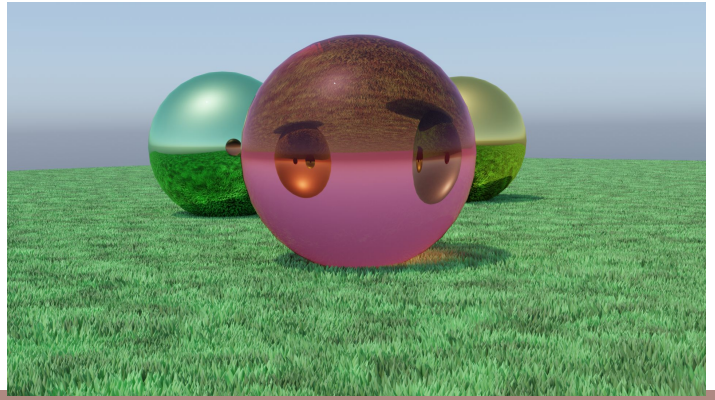

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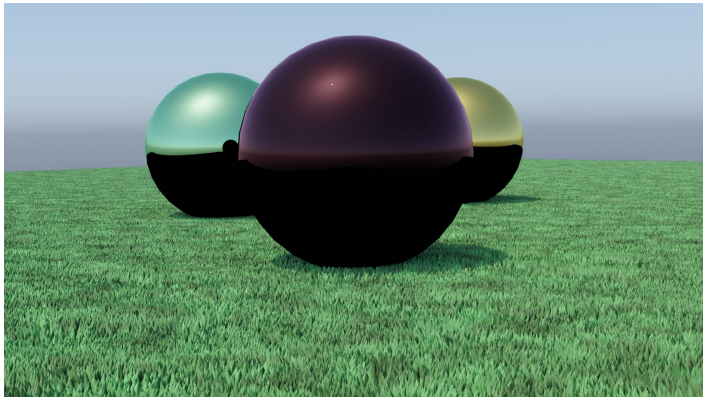

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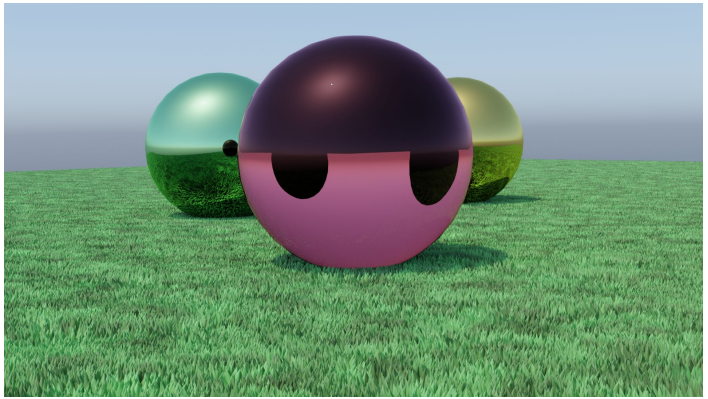

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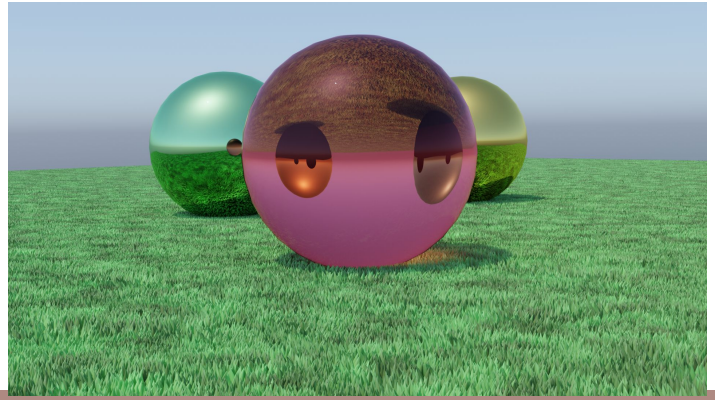

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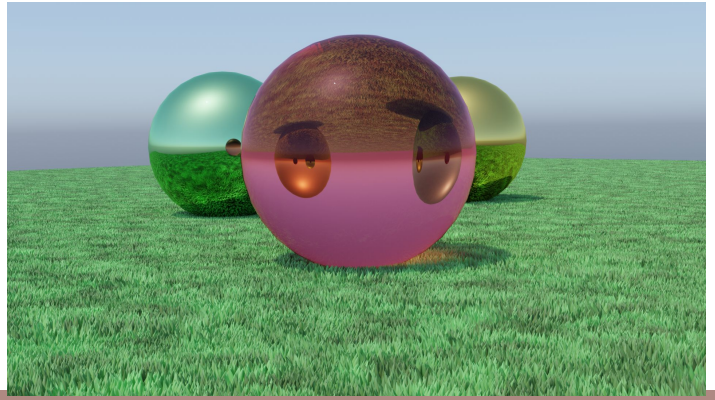

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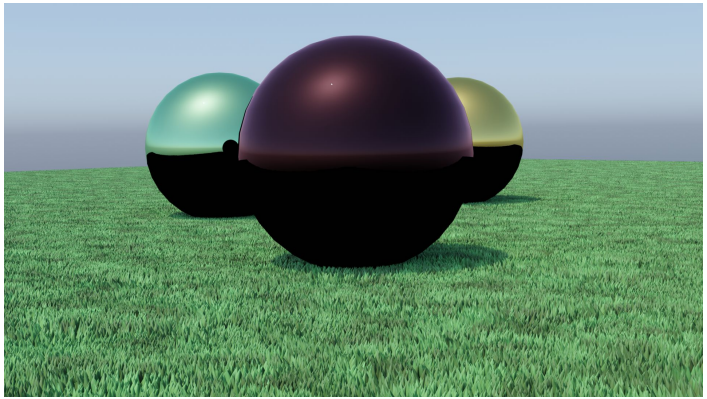
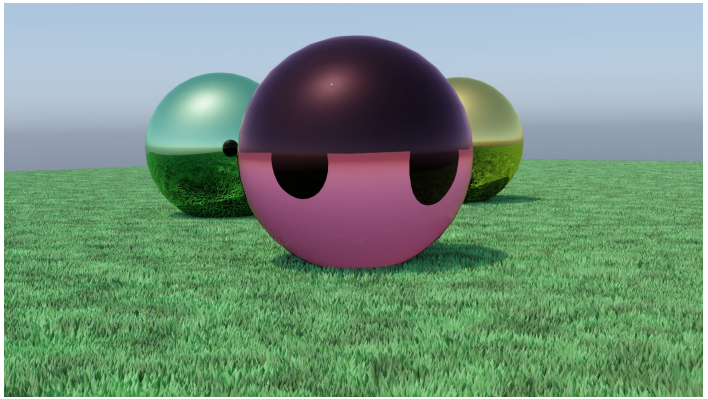
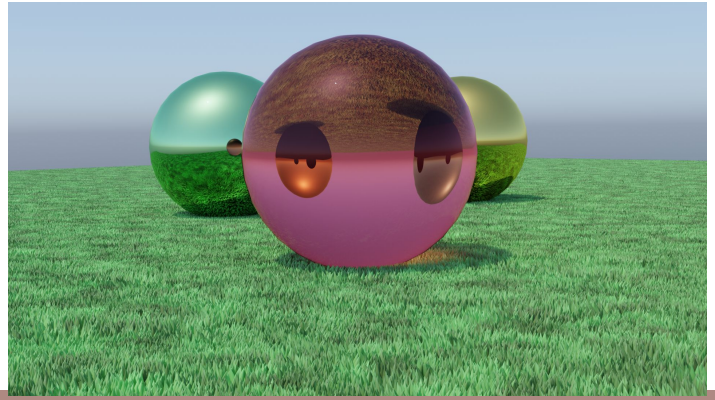


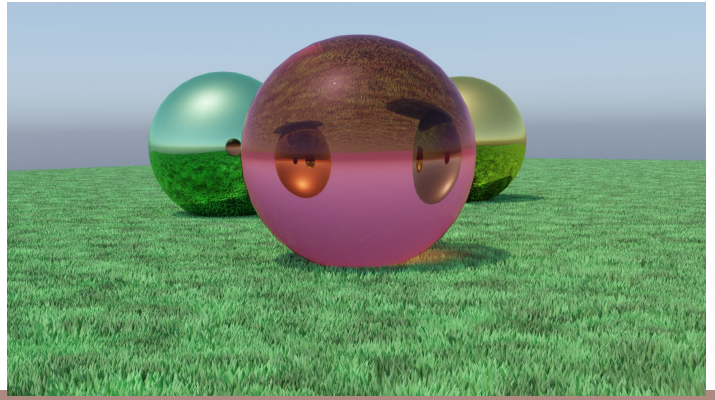
1 recursion bounce



2 recursion bounces

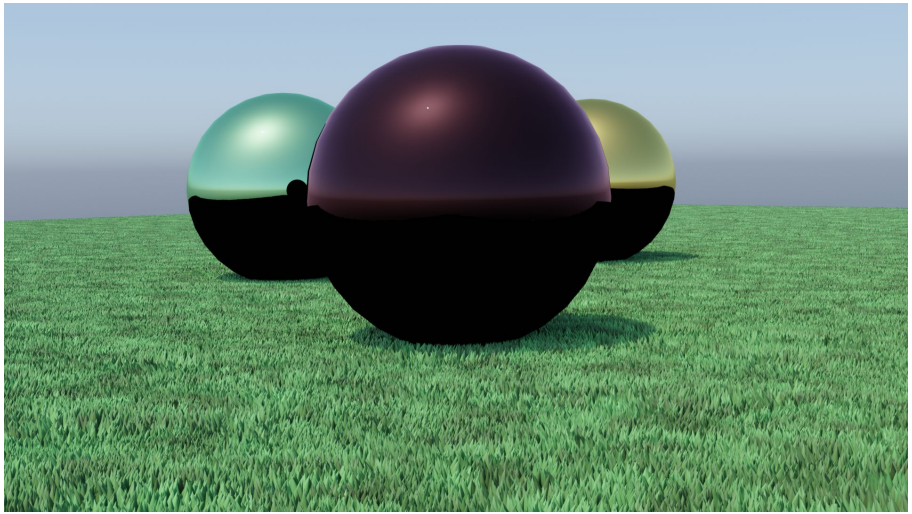


3 recursion bounces

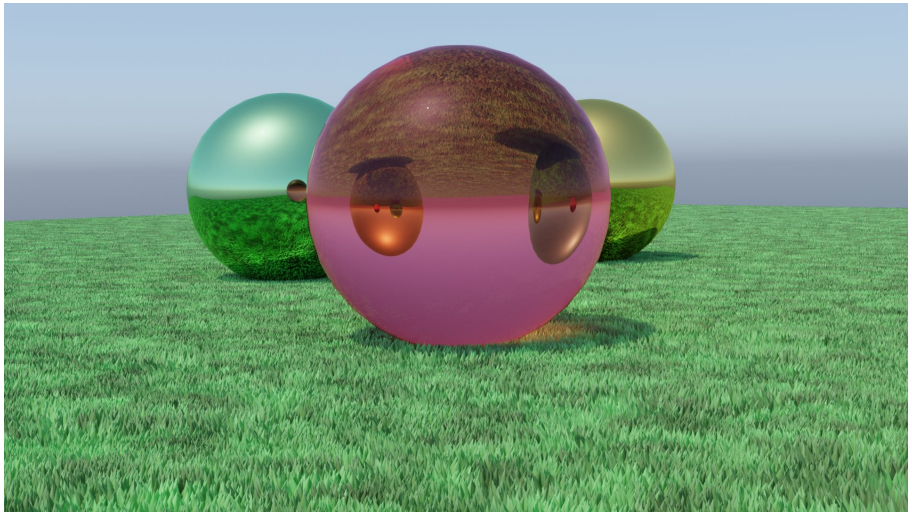


Recursive Raytracing: Termination

Max bounces = 0



Max bounces = 16

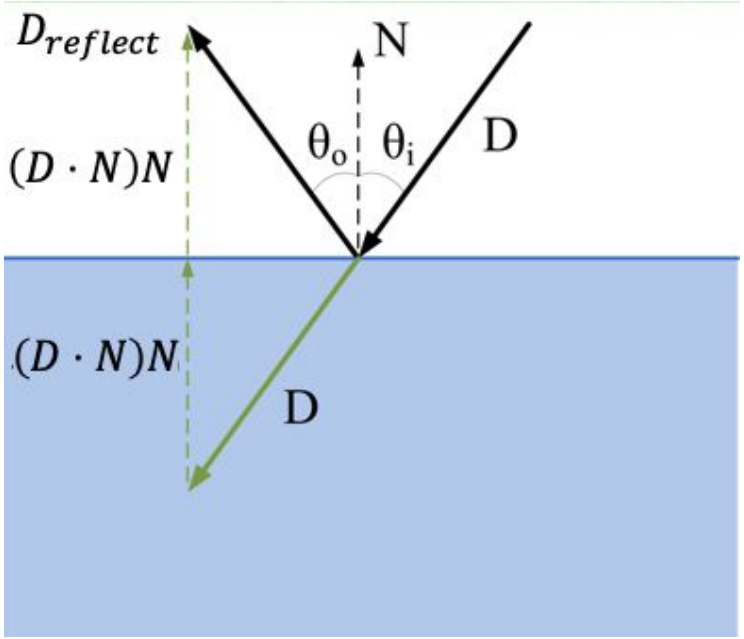


Lecture Outline

- Ray tracing review
- Raycasting & recursive raytracing
- Reflected & transmitted rays
  - Refraction
  - Total internal reflection
- Attenuation & caustics

Reflected Rays

- Reflected rays bounce off of an object at the same angle the ray arrived



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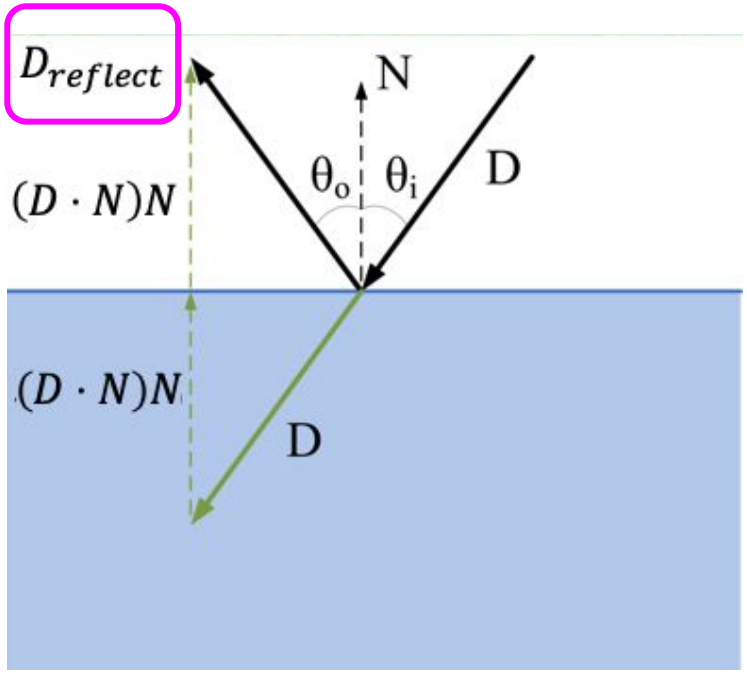

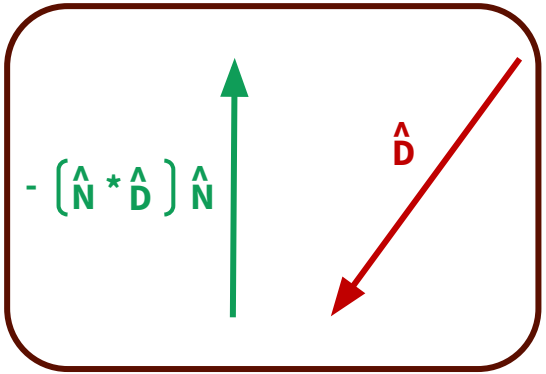

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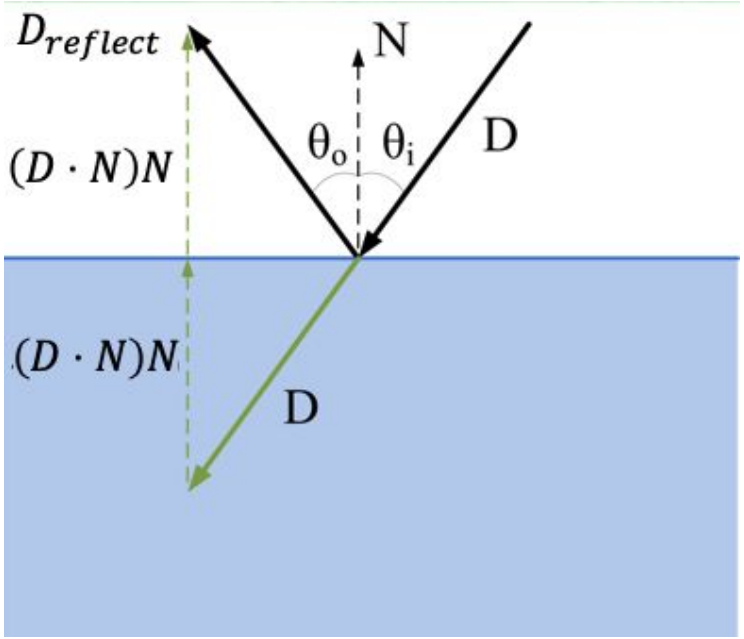
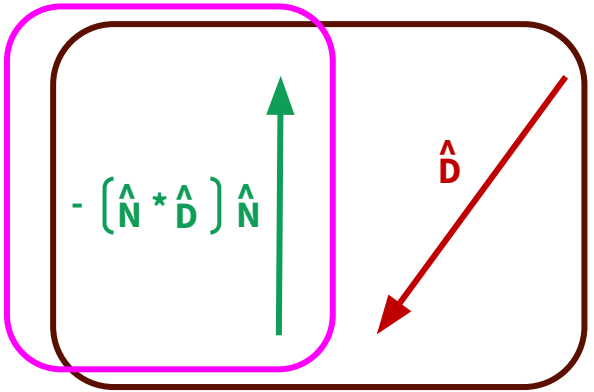


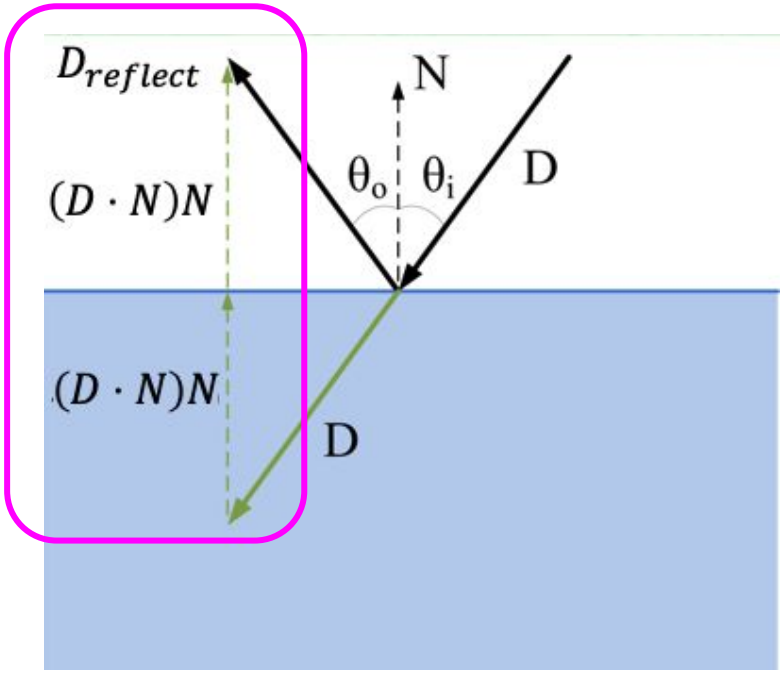
Decompose *D* into parallel and perpendicular (to *N*) components



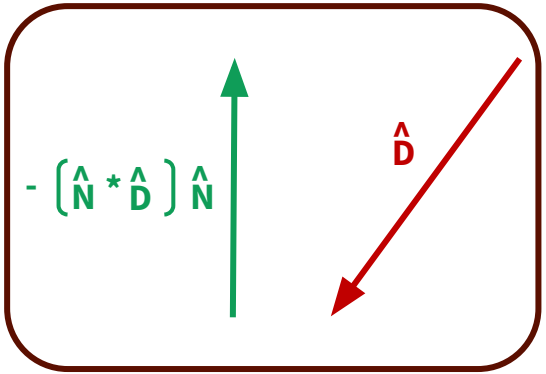
$$\hat{D}_{reflect} = \left(\hat{N} \cdot \hat{D}\right)\hat{N}$$

Reflected Rays

- Reflected rays bounce off of an object at the same angle the ray arrived



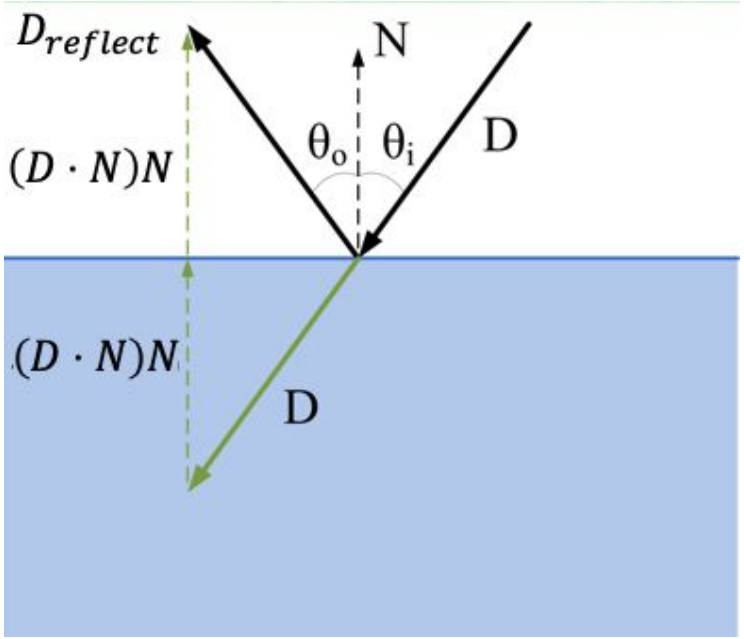
Component of *D* perpendicular to *N* stays the same, parallel component gets flipped



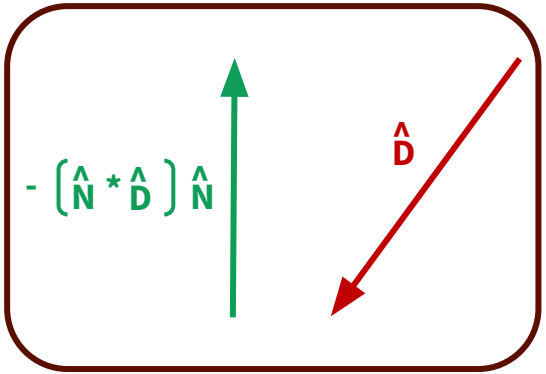
$$\hat{D}_{reflect} = \hat{D} - \left(\hat{N} \cdot \hat{D}\right)\hat{N}$$

Reflected Rays

- Reflected rays bounce off of an object at the same angle the ray arrived



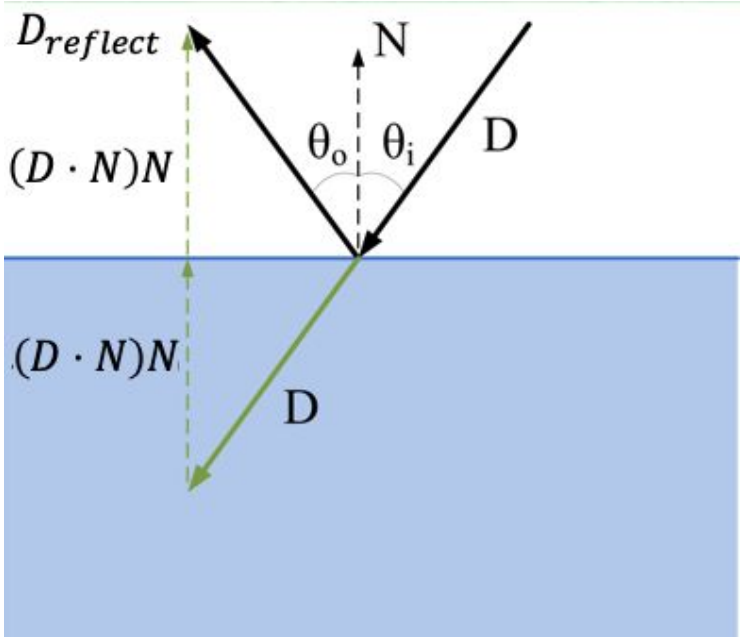
Subtract the parallel to *N* part of *D* twice; first to cancel the inward component, second to build the outward one

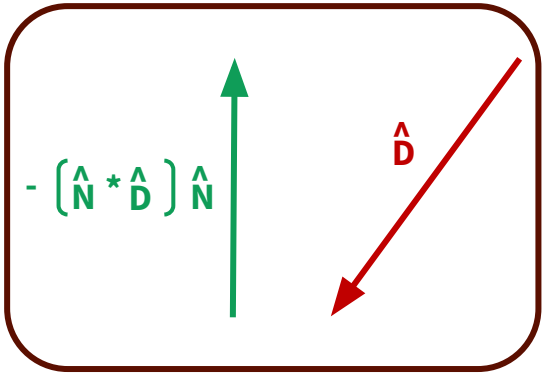


$$\hat{D}_{reflect} = \hat{D} - 2\left(\hat{N} \cdot \hat{D}\right)\hat{N}$$

Reflected Rays

- Reflected rays bounce off of an object at the same angle the ray arrived

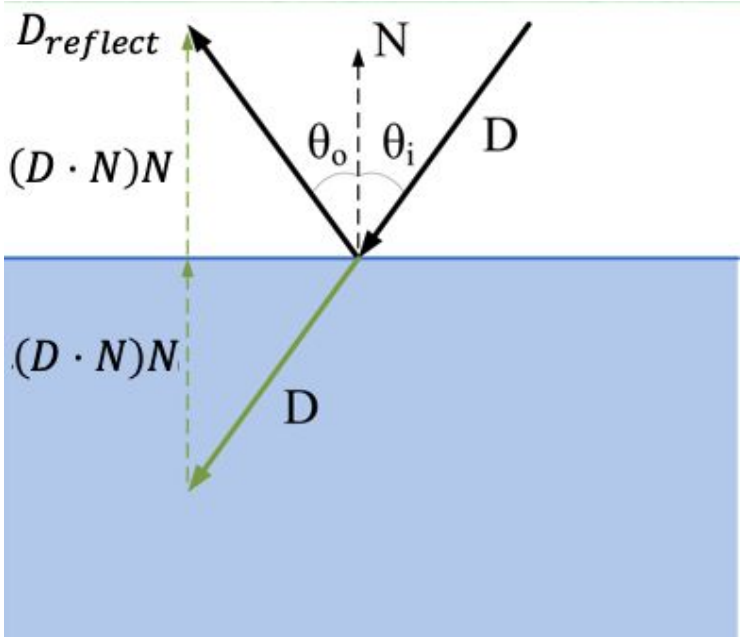


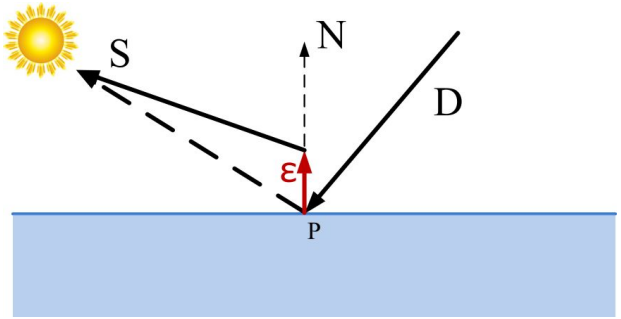


$$\hat{D}_{reflect} = \hat{D} - 2\left(\hat{N} \cdot \hat{D}\right)\hat{N}$$

Reflected Rays

- Recall: spurious self-occlusion!



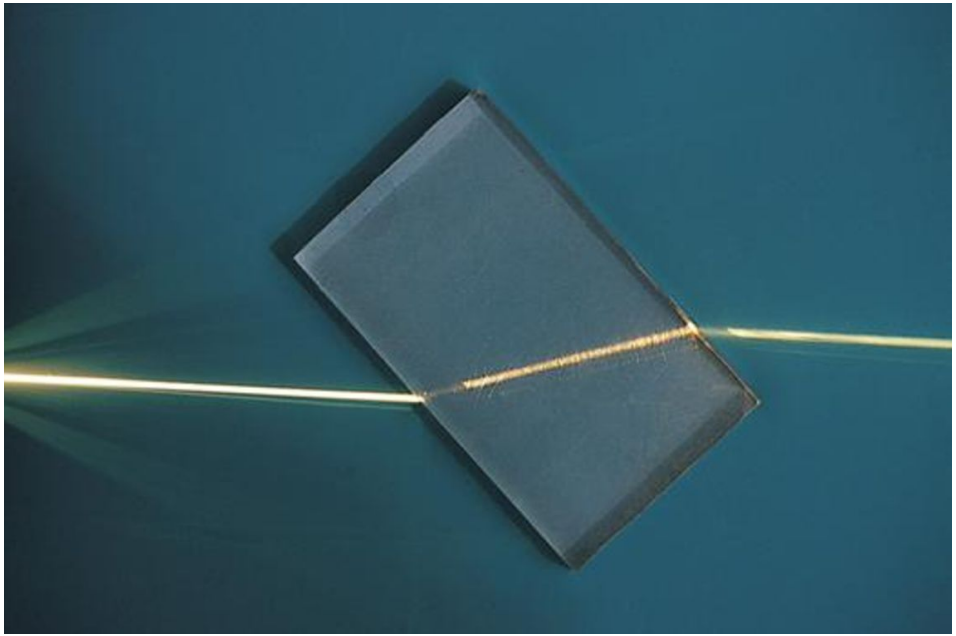


$$\hat{D}_{reflect} = \hat{D} - 2\left(\hat{N} \cdot \hat{D}\right)\hat{N}$$

$$R_{reflect}(t) = \left(P + \epsilon\hat{N}\right) + \hat{D}_{reflect}\, t$$

Transmitted Rays

- Light bends as it passed through **different materials**



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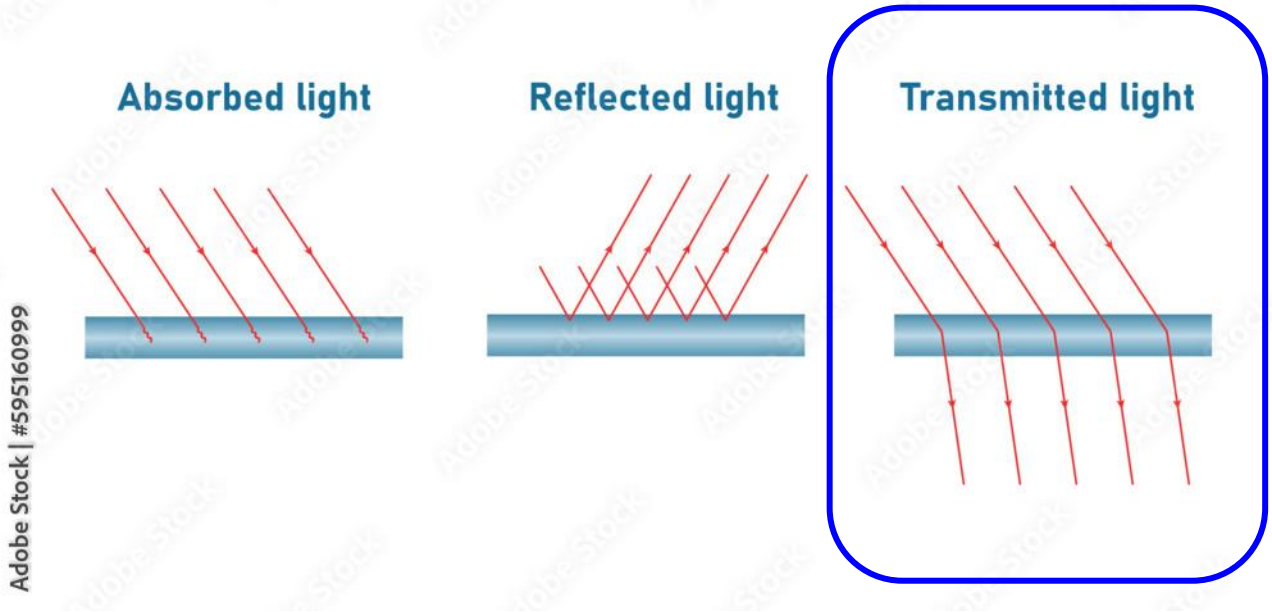

Transmitted Rays

- Light bends as it passed through different materials
- **Snell's law** ⇒ gives the relationship between angle of incidence and transmittance
- $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$ where n1 the index of refraction for the first medium

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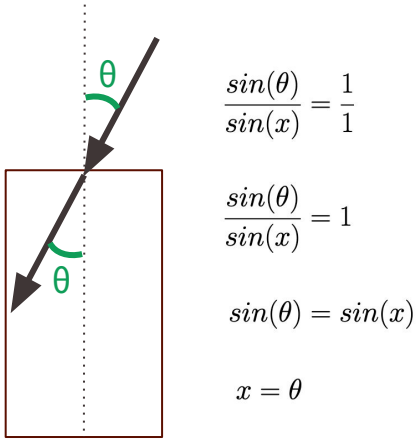
Material	Index of Refraction (n)
Vacuum	1.000
Air	1.000277
Water	1.333333
Ice	1.31
Glass	About 1.5
Diamond	2.417

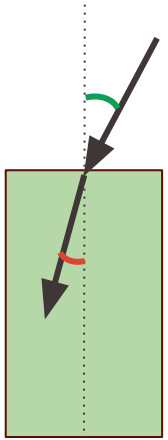


$$\frac{sin(\theta)}{sin(x)} = \frac{1}{1}$$

$$\frac{sin(\theta)}{sin(x)} = 1$$

$$sin(\theta) = sin(x)$$

$$x = \theta$$



$$\frac{sin(\theta)}{sin(x)} = \frac{1.5}{1}$$

$$\frac{sin(\theta)}{sin(x)} = 1.5$$

$$sin(x) = \frac{sin(\theta)}{1.5}$$

$$x = sin^{-1}(\frac{sin(\theta)}{1.5})$$

Transmitted Rays

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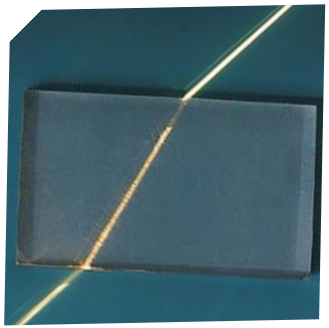

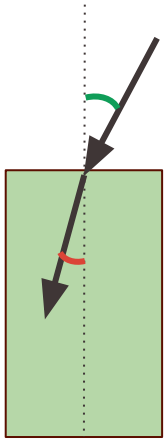

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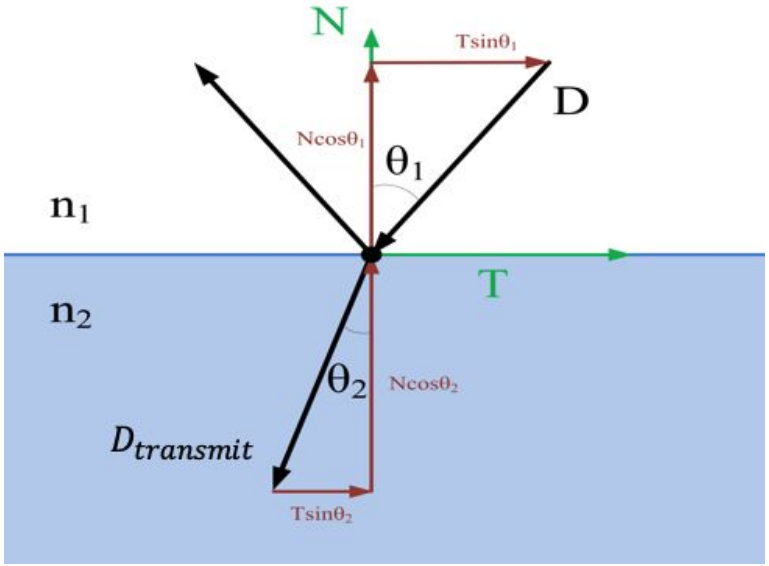
Transmitted Rays

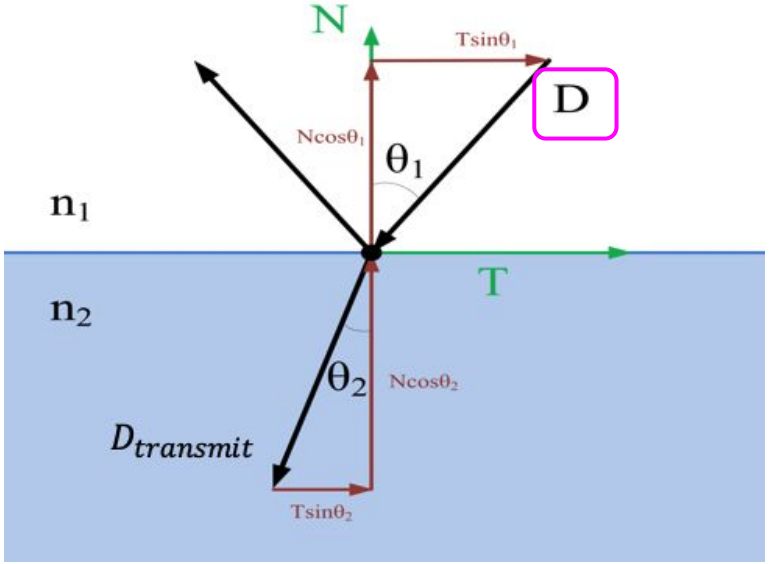
- We want to represent a transmitted ray in terms of n1, n2, D, N



Transmitted Rays

- We want to represent a transmitted ray in terms of n1, n2, D, N

$$\hat{D} = -\cos\theta_1 \hat{N} - \sin\theta_1 \hat{T}$$



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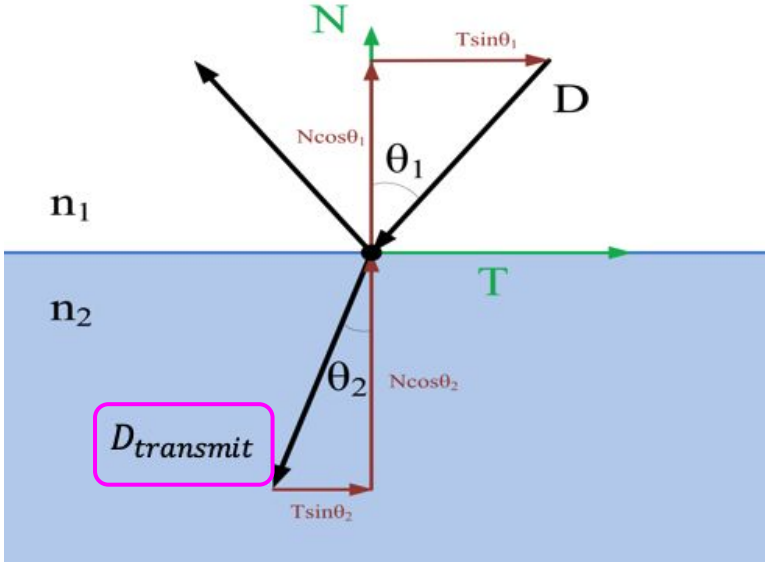
$$\hat{D}_{transmit} = -\cos\theta_2 \hat{N} - \sin\theta_2 \hat{T}$$



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$$\hat{D} = -\cos\theta_1 \hat{N} - \sin\theta_1 \hat{T}$$

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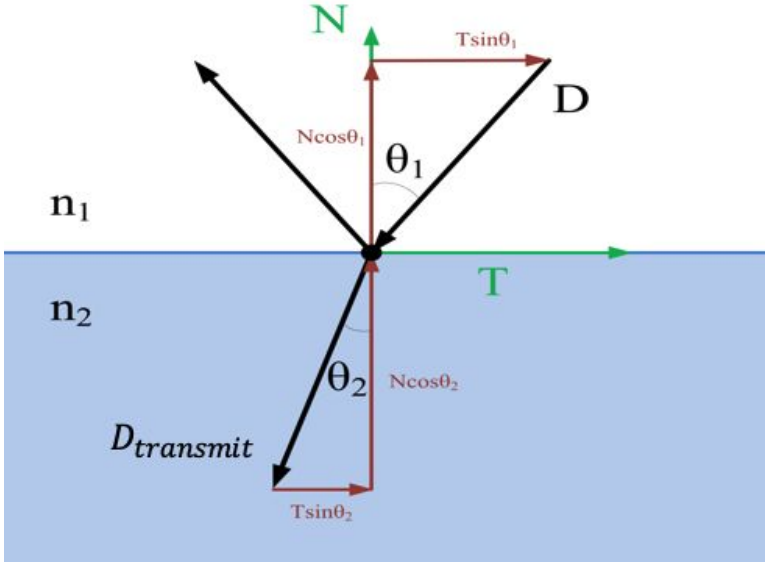
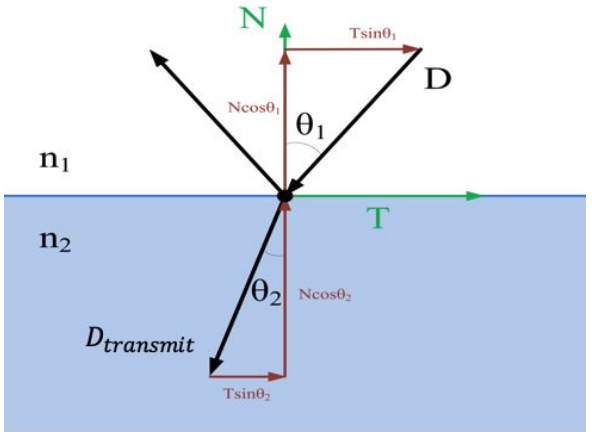
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$



Transmitted Rays

$$\hat{D}_{transmit} = -\cos\theta_2 \hat{N} - \sin\theta_2 \hat{T}$$



- So $\hat{D}_{transmit} = -\hat{T}sin\theta_2 - \hat{N}cos\theta_2 = (\hat{D} + \hat{N}cos\theta_1)\frac{sin\theta_2}{sin\theta_1} - \hat{N}\sqrt{1 - \sin^2\theta_2}$
- Using Snell's Law, $\hat{D}_{transmit} = (\hat{D} + \hat{N}cos\theta_1)\frac{n_1}{n_2} - \hat{N}\sqrt{1 - \left(\frac{n_1}{n_2}sin\theta_1\right)^2}$
- $\hat{D}_{transmit} = \hat{D}\frac{n_1}{n_2} + \hat{N}\left(\frac{n_1}{n_2}cos\theta_1 - \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2(1 - \cos^2\theta_1)}\right)$
- Using $cos\theta_1 = -\hat{D}\cdot\hat{N}$ gives $\hat{D}_{transmit} = \hat{D}\frac{n_1}{n_2} - \hat{N}\left(\frac{n_1}{n_2}\hat{D}\cdot\hat{N} + \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2\left(1 - (\hat{D}\cdot\hat{N})^2\right)}\right)$

$$\hat{D} = -\cos\theta_1 \hat{N} - \sin\theta_1 \hat{T}$$

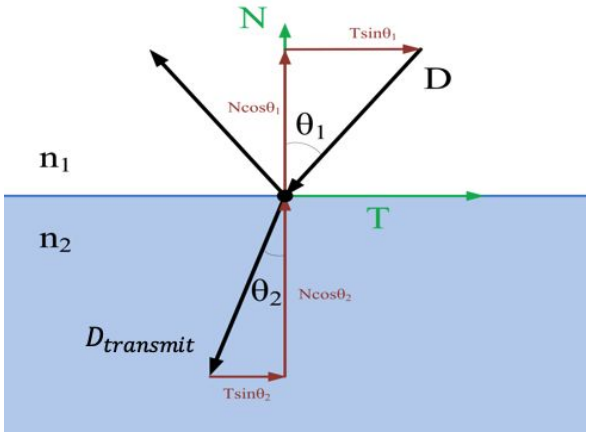
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\cos\theta_1 = -\hat{N}\cdot\hat{D}$$

$$\hat{D}_{transmit} = \frac{n_1}{n_2}\hat{D} - \left(\frac{n_1}{n_2}\hat{N}\cdot\hat{D} + \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2\left(1 - \left(\hat{N}\cdot\hat{D}\right)^2\right)}\right)\hat{N}$$

Transmitted Rays

$$\hat{D}_{transmit} = -\cos\theta_2 \hat{N} - \sin\theta_2 \hat{T}$$



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$$\hat{D} = -\cos\theta_1 \hat{N} - \sin\theta_1 \hat{T}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\cos\theta_1 = -\hat{N}\cdot\hat{D}$$

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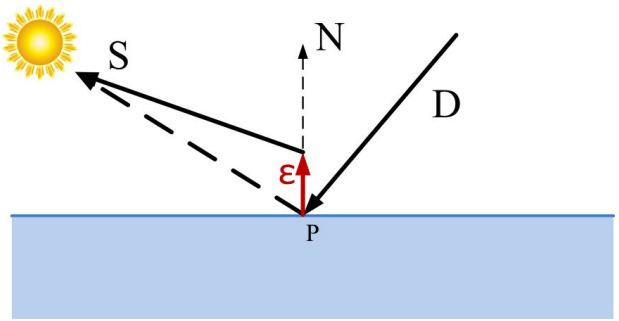
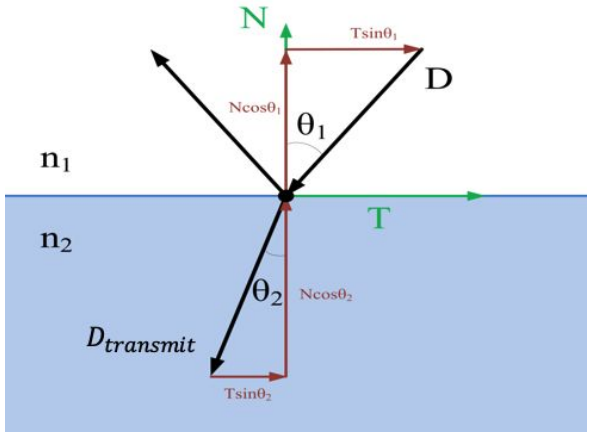
Transmitted Rays

- Don’t forget spurious self-intersection (flip the normals)

$$\hat{D}_{transmit} = \frac{n_1}{n_2}\hat{D} - \left(\frac{n_1}{n_2}\hat{N}\cdot\hat{D} + \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\left(1-\left(\hat{N}\cdot\hat{D}\right)^2\right)}\right)\hat{N}$$

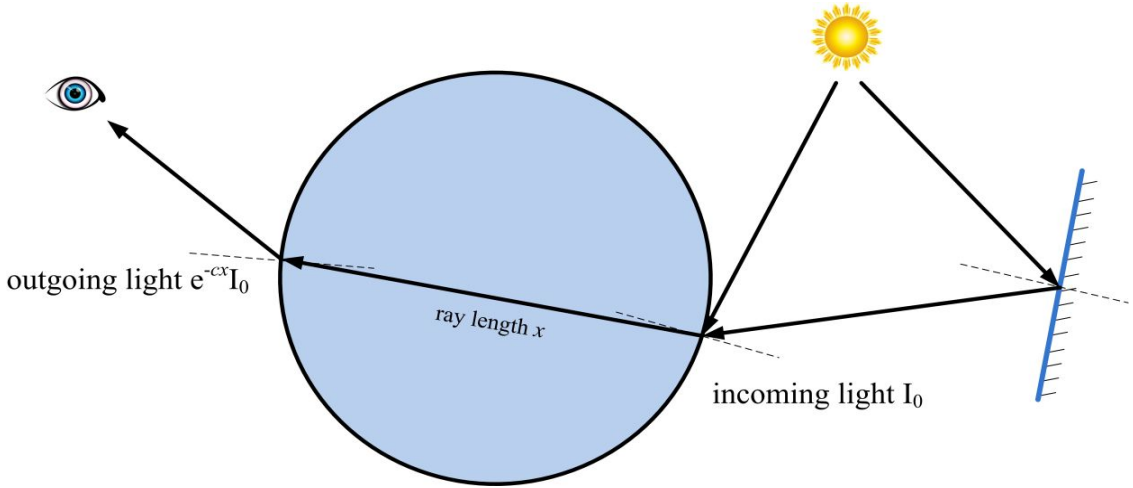
$$R_{transmit}(t) = \left(P - \epsilon\hat{N}\right) + \hat{D}_{transmit}\,t$$





Refraction

- Transmission: compute snell's law twice (once for entering, once for exiting)
- When computing intersection from inside an object, **flip the normals**



Refraction

function trace(ray, objects, lights):

intersection=ray_geometry_intersection(ray, objects)

color=**compute_BRDF_at_intersection**(intersection, objects, lights)

if (material at intersection has reflectivity>0)

reflected_ray=create_reflected_ray(ray, intersection)

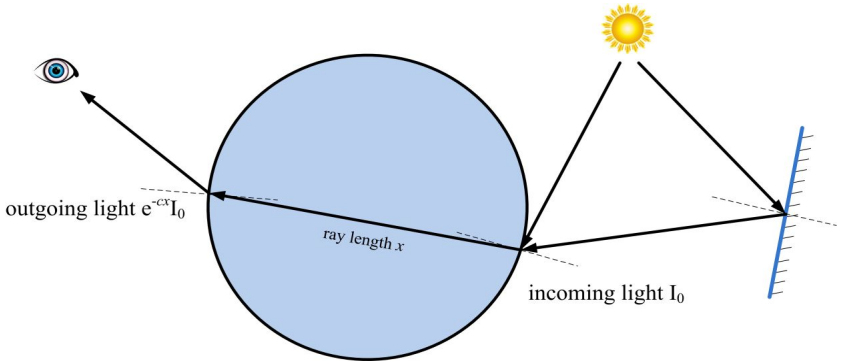
color=color+**trace(reflected_ray,objects,lights)*reflectivity**

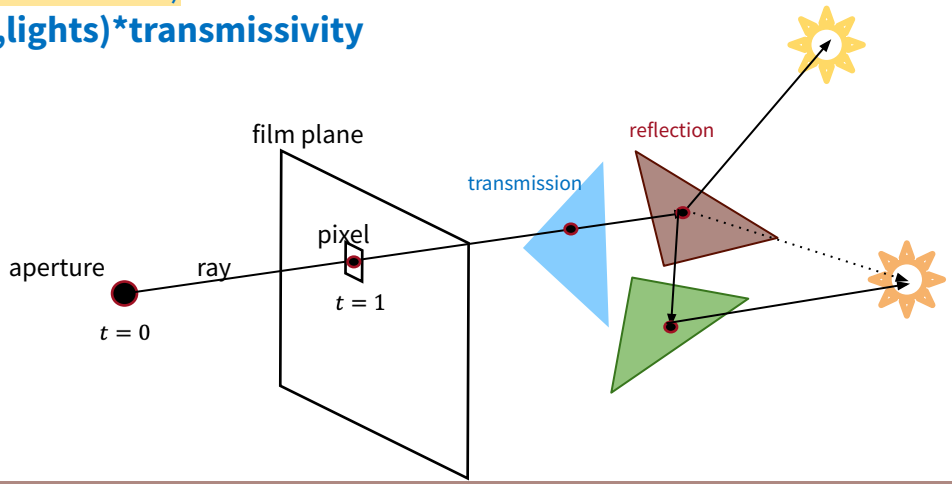
if (material at intersection has transmissivity>0)

refracted_ray=create _refracted_ray(ray, intersection) **<- compute exiting ray**

color=color+**trace(refracted_ray,objects,lights)*transmissivity**

return color





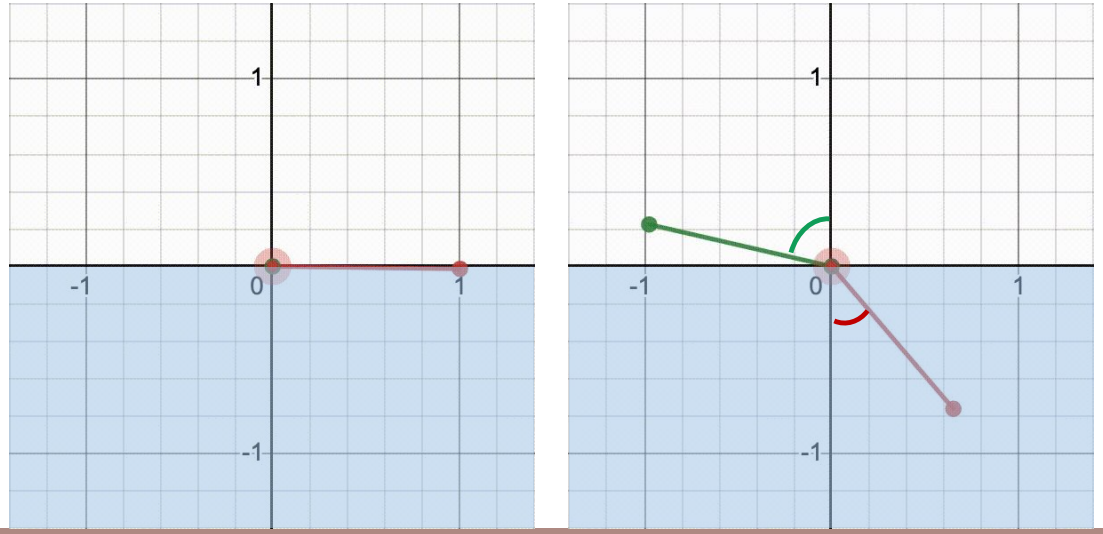
Total Internal Reflection

- Occurs when the term under the square root is negative
- No transmission – instead, we have reflection on the other side of the object

$$\hat{D}_{transmit} = -\frac{n_1}{n_2}\hat{D} - \left(\frac{n_1}{n_2}\hat{N}\cdot\hat{D} + \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\left(1-\left(\hat{N}\cdot\hat{D}\right)^2\right)}\right)\hat{N}$$

Total Internal Reflection

- Occurs when we go from a higher index of refraction (IOR) to a lower IOR
  - We use the boundary angle to determine this (the **critical angle**)



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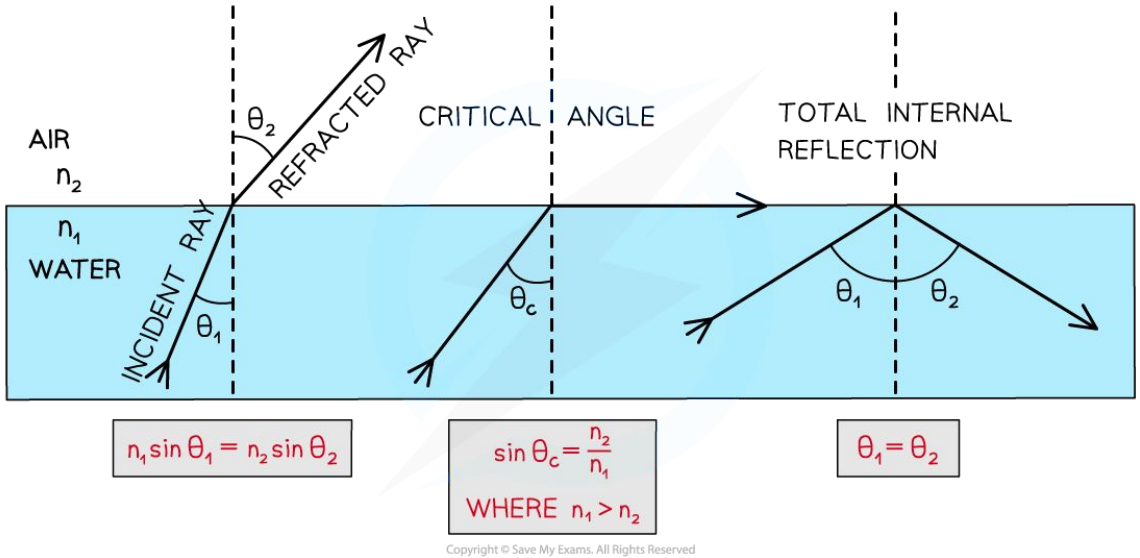
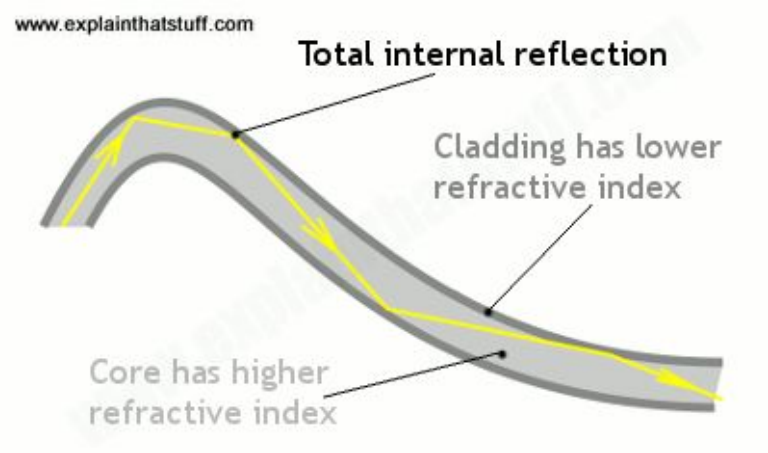
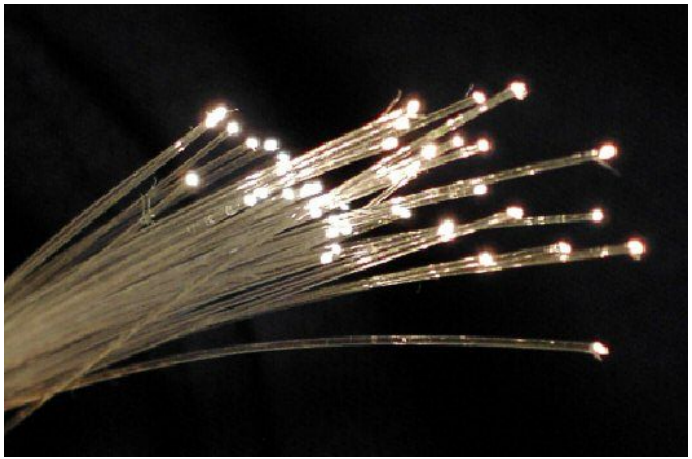


Total Internal Reflection

- Occurs when we go from a higher index of refraction (IOR) to a lower IOR
  - We use the boundary angle to determine this (the **critical angle**)





Reflection vs Transmission

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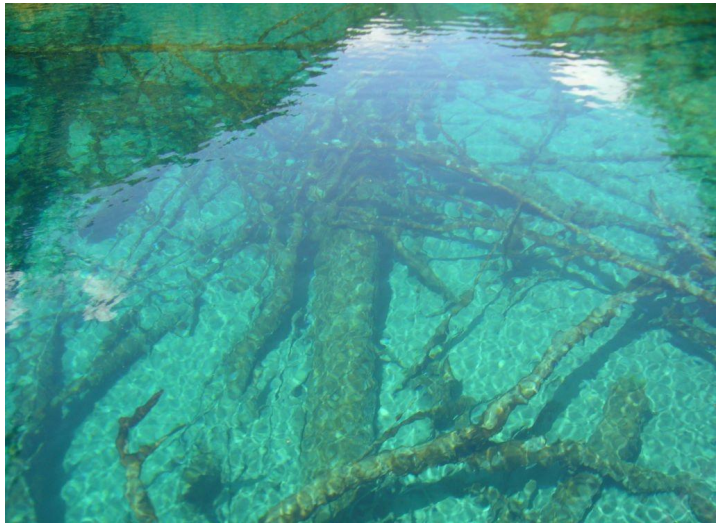

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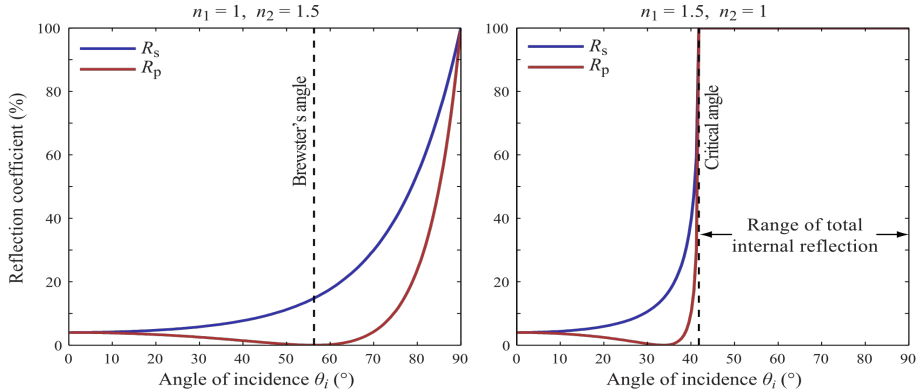
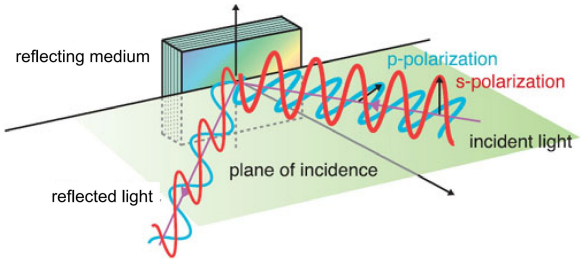
Reflection vs Transmission

- Amount of transmission vs. reflection decreases as the viewing angle goes from perpendicular (overhead) to parallel (grazing)
  - This is determined by the **Fresnel equations**
- Interesting detail: light is polarized and behaves differently based on its orientation relative to the incident plane



Light entering a denser material (e.g. from air into water)



Reflection vs Transmission

- Light is polarized – behaves differently based on orientation relative to incident plane

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2 \qquad R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

- Transmission ⇒ usually amount of light that is not reflected

$$T_p = 1 - R_p \qquad T_s = 1 - R_s$$

- In rendering, we usually just take the average and treat light as unpolarized

$$R = \frac{R_p + R_s}{2} \qquad T = 1 - R$$

Schlick's Approximation

$$R(\theta) = R_0 + (1 - R_0)(1 - \cos\theta)^5$$

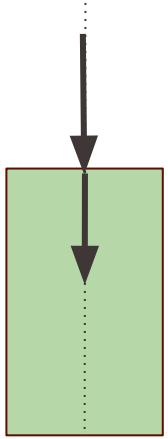
where

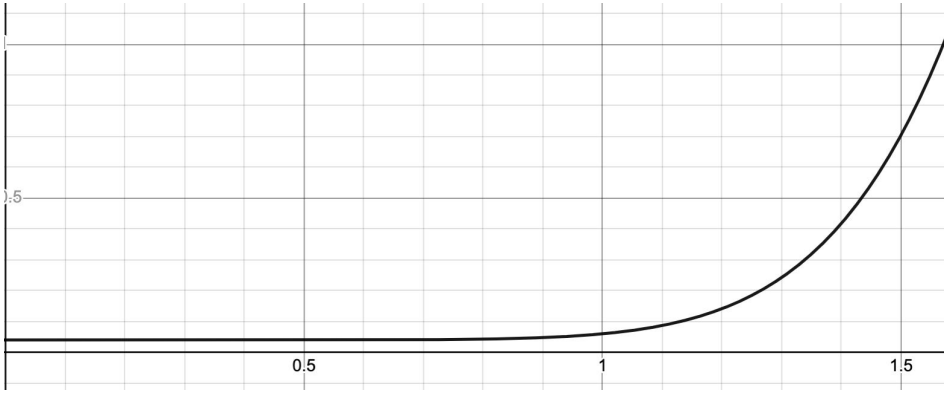
$$R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$n_1 = 1.0$

$n_2 = 1.5$

$R_0 = .04$





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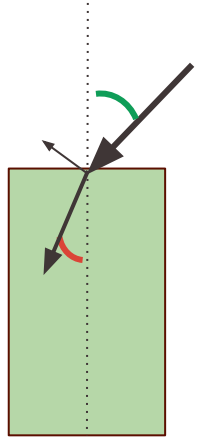


Lecture Outline

- Ray tracing review
- Raycasting & recursive raytracing
- Reflected & transmitted rays
  - Refraction
  - Total internal reflection
- Attenuation & caustics

Attenuation

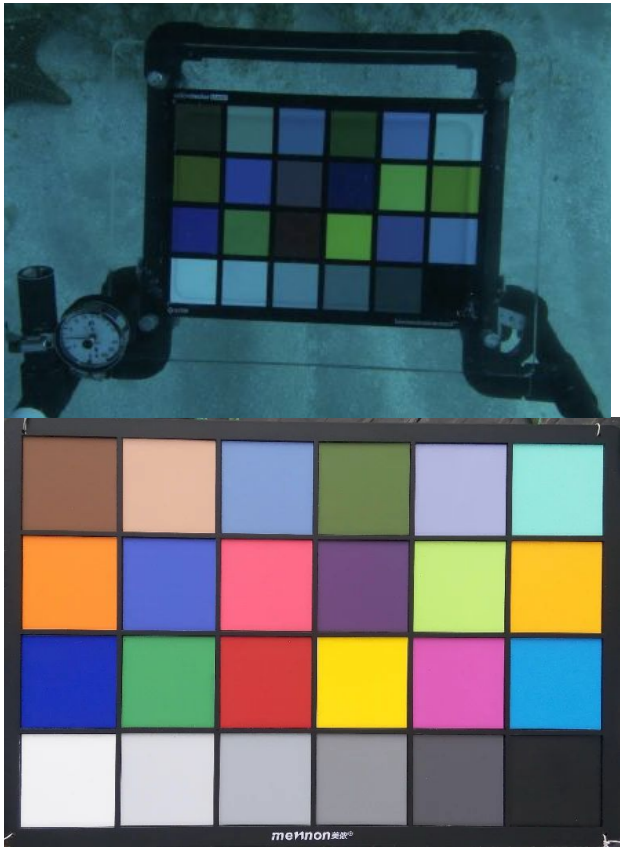
- When light is transmitted through objects, some light may get absorbed

Attenuation

- When light is transmitted through objects, some light may get absorbed
- Examples:
  - Shallow water is clear (almost no attenuation)
  - Deeper water attenuates all the red light and looks bluish-green
  - Even deeper water attenuates all the green light too, and looks blue
  - Eventually all the light attenuates, and the color ranges from blackish-blue to black

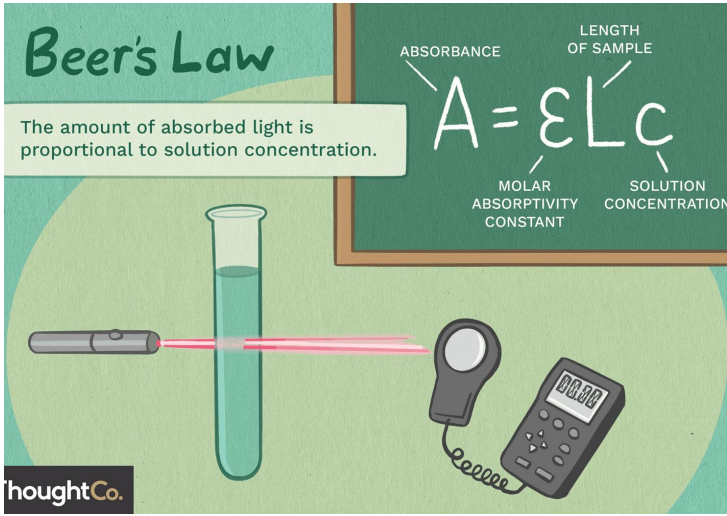


Attenuation



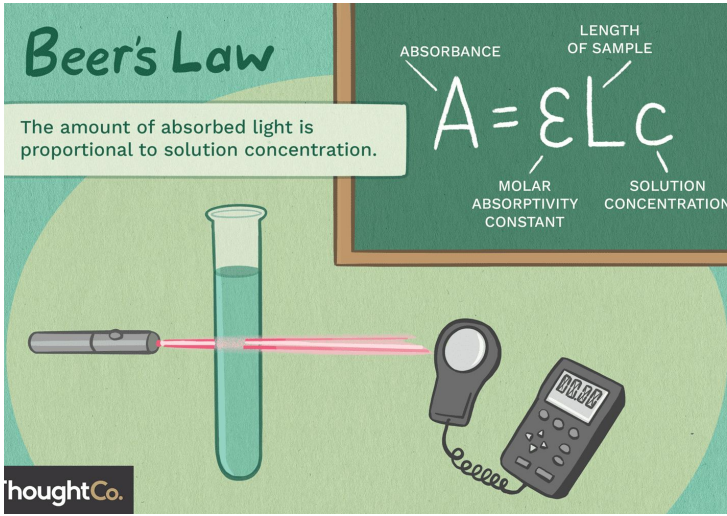
Attenuation

- **Beer's law** ⇒ describes the relationship between the absorbance of light by a solution and its concentration



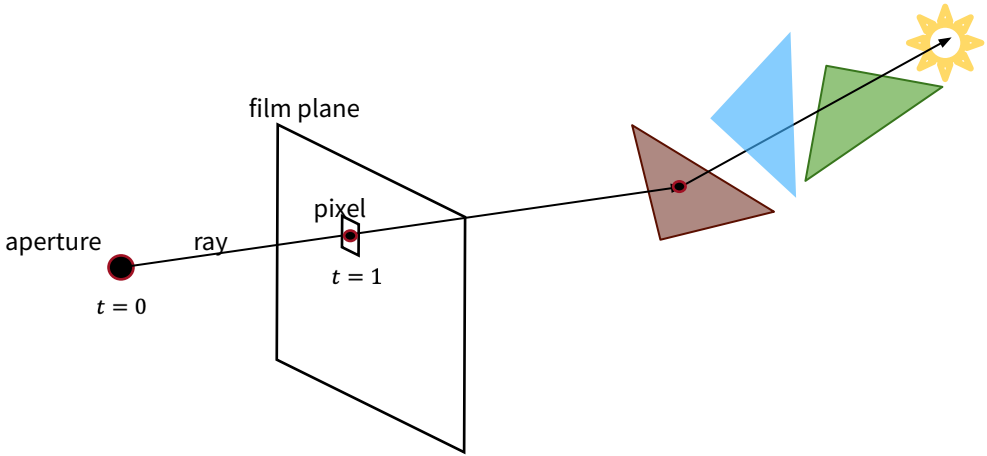
Attenuation

- **Beer's law** ⇒ describes the relationship between the absorbance of light by a solution and its concentration
  - Chemistry: given what the light looks like, what is the material
  - Graphics: given the material, what does the light look like through it



Attenuation: Shadow Rays

- Attenuation can approximate shadows of translucent objects
- Assuming the media is **homogeneous**, attenuation along a ray can be described by Beer's law



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  - Therefore, we can’t just use straight line shadow rays to each light course

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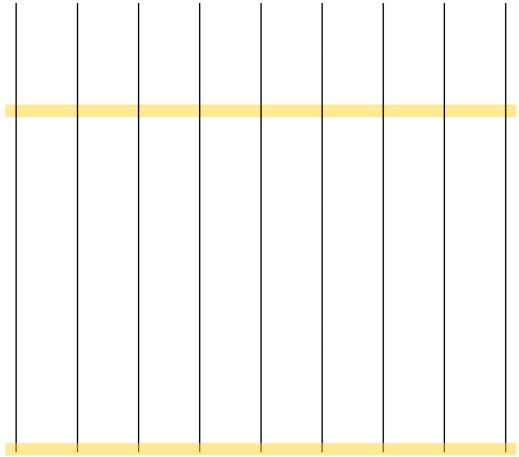

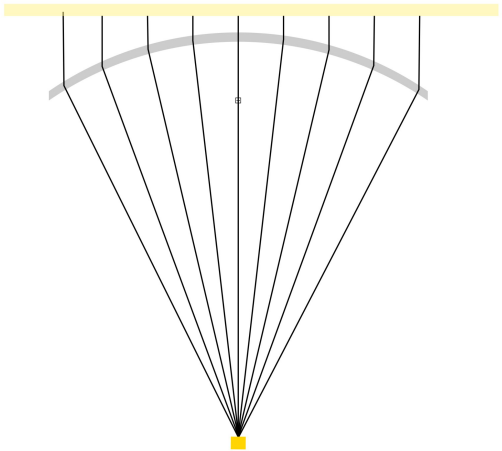

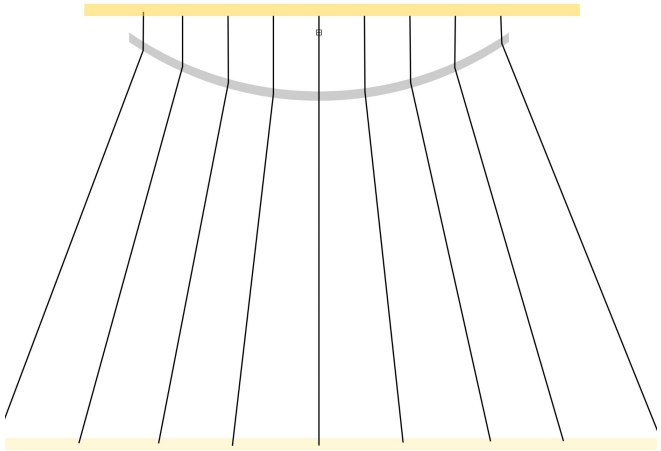

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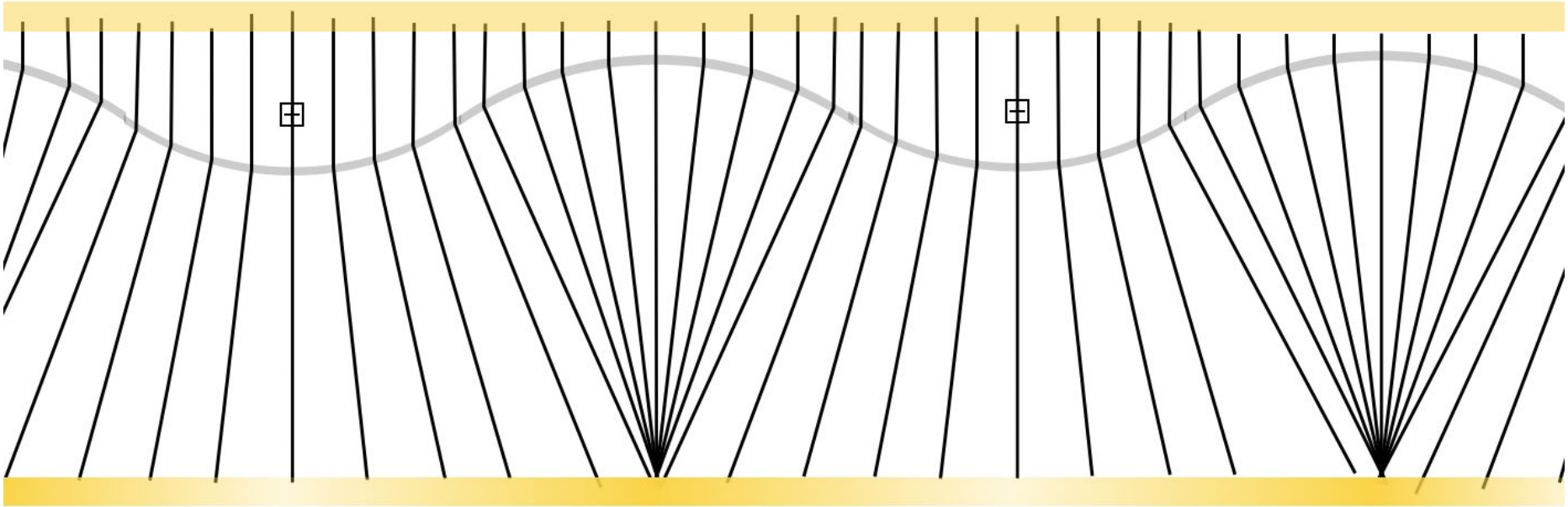

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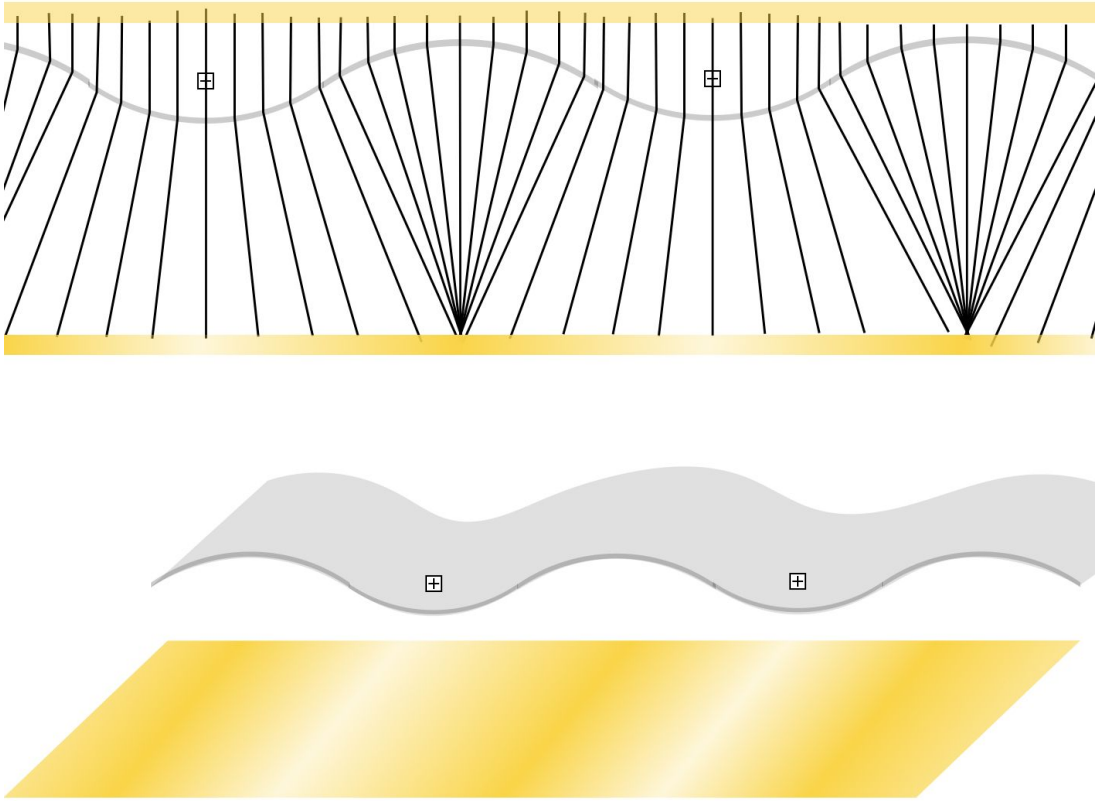

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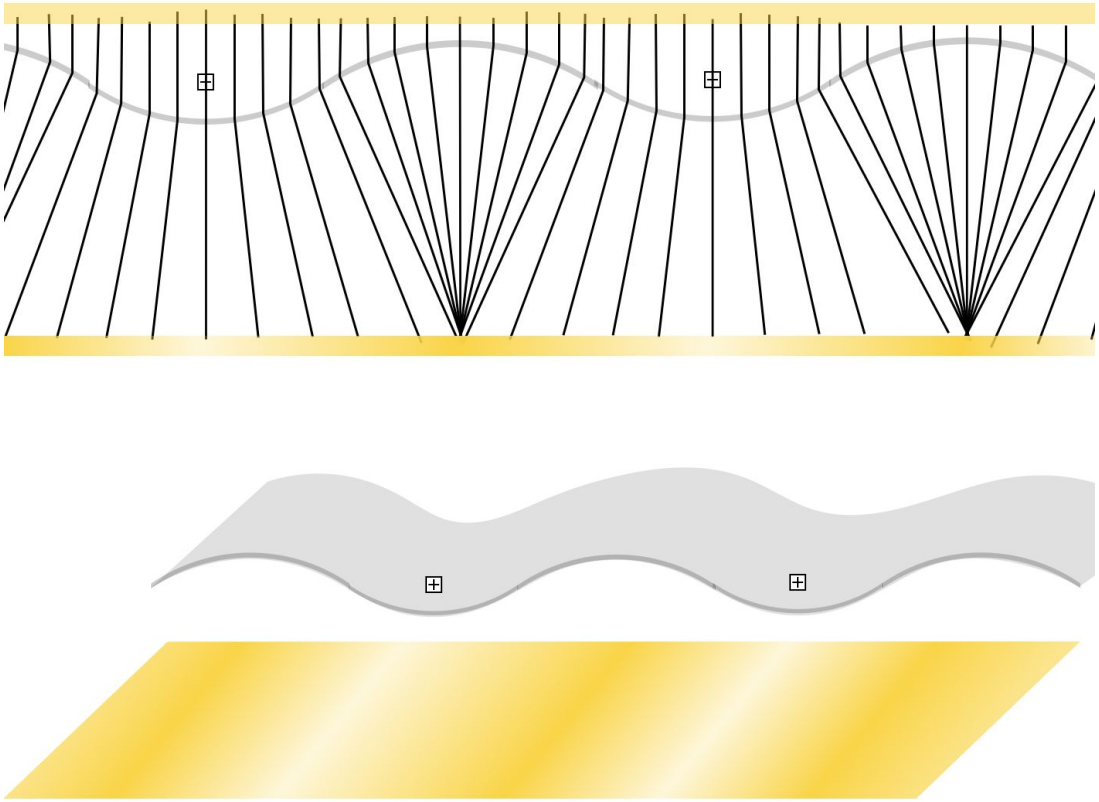

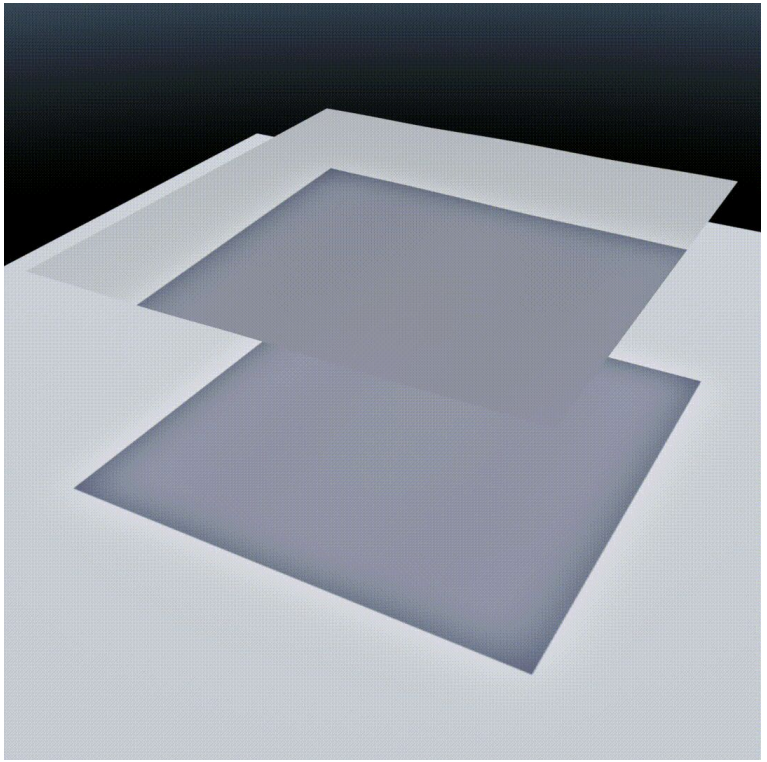

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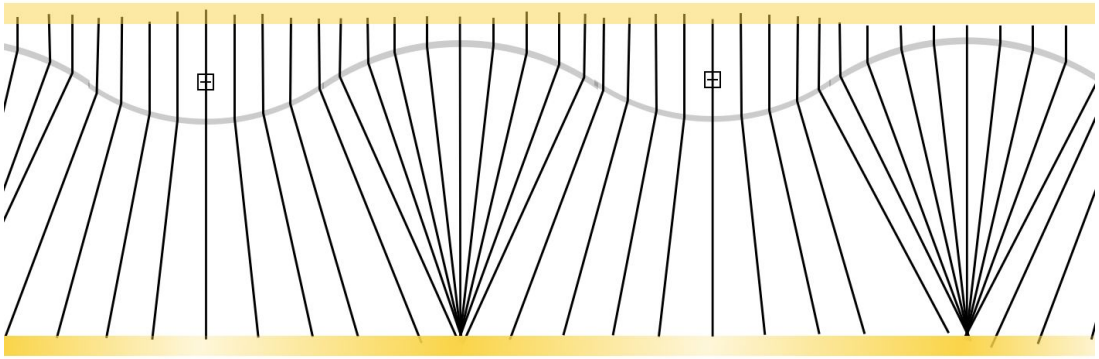

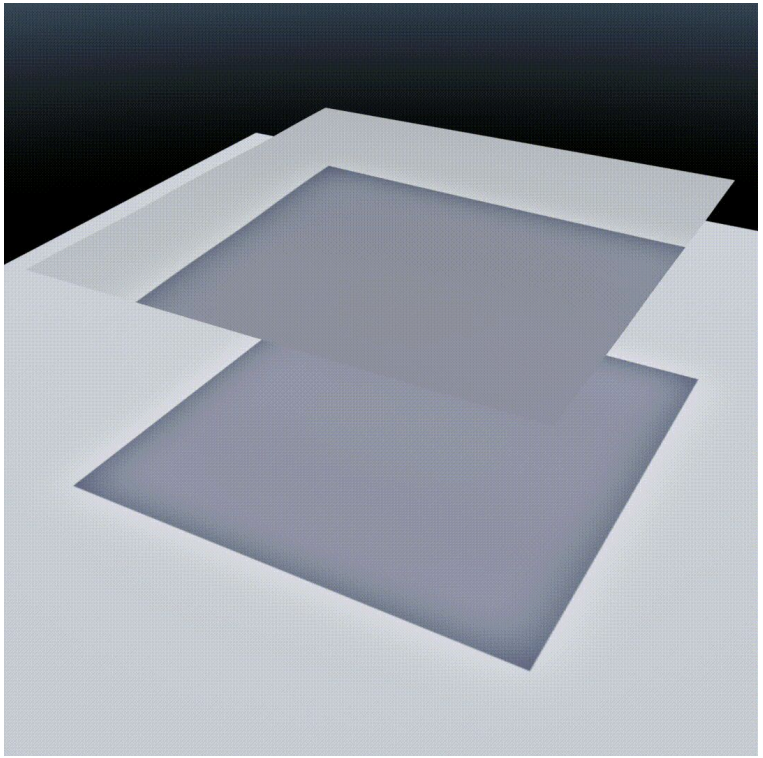

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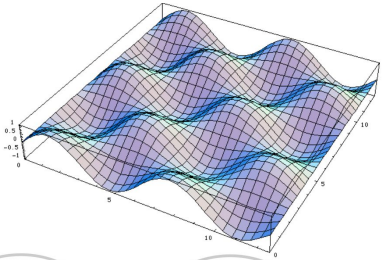

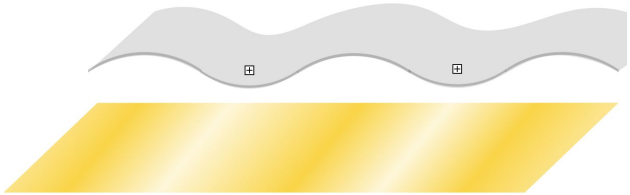

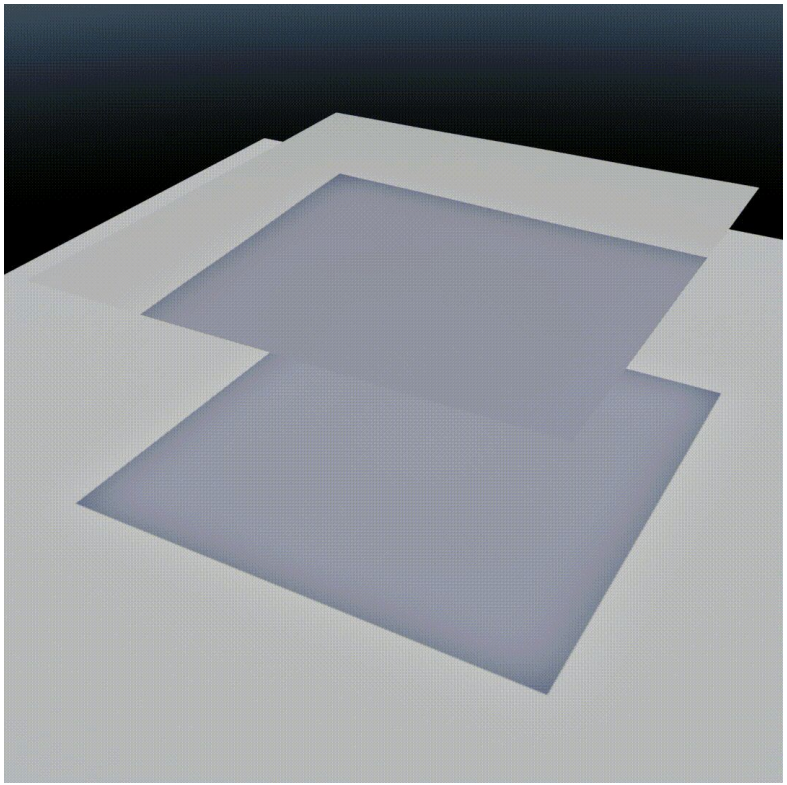

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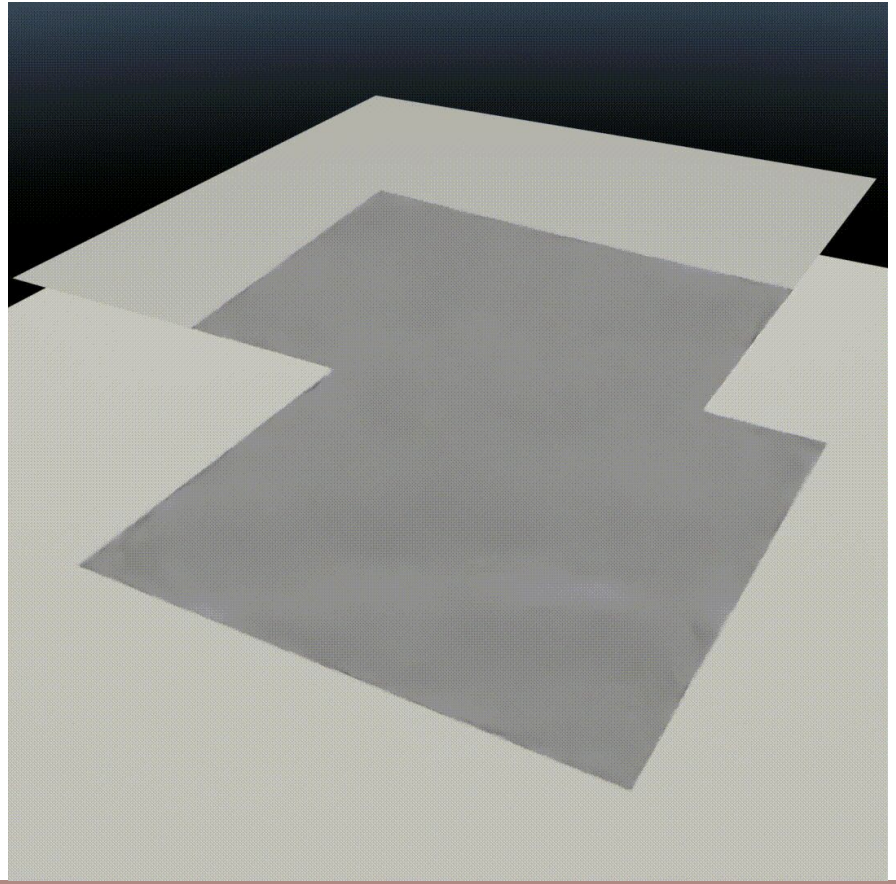

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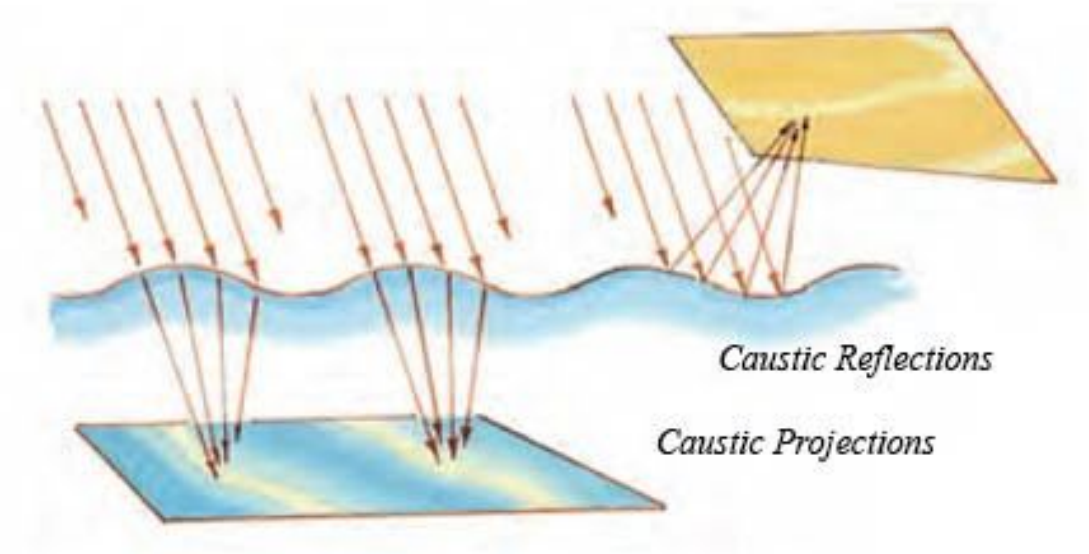

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  - Find the closest intersection point with an object in the scene and this ray
  - Send out more rays (shadow, reflected, transmissive)
    - With computer error correction
    - And considering material properties
- Think about making this cheap for the computer while maintaining high fidelity
  - Limiting number of rays (depth)
  - Prioritizing objects over one another for raytracing

Additional Resources + Exploring!

- ExplainThatStuff: Fiber Optics
- Fresnel Equations Derivations and Explanations
- Polarization
- Schlick's Approximation
- Underwater Image Restoration via Maximum Attenuation Identification
- Caustic light patterns inspire new glass artwork

- Fundamentals of Computer Graphics, Steve Marschner and Peter Shirley
  - Preview, Book